CONSTRAINED TENSOR DECOMPOSITIONS
POLARIMETRIC TIME SERIES CHANGE ANALYSIS

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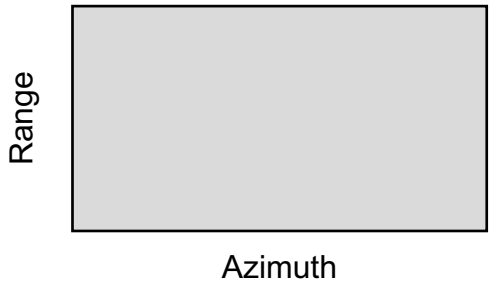
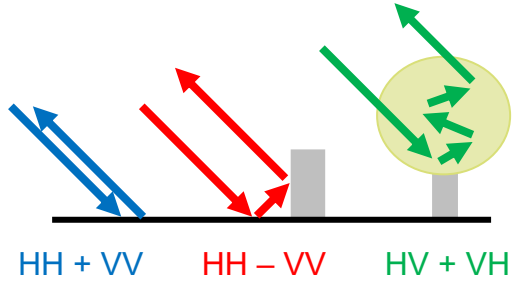
Multidimensional Data

Space

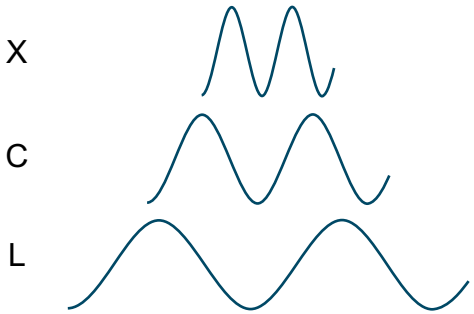


Time

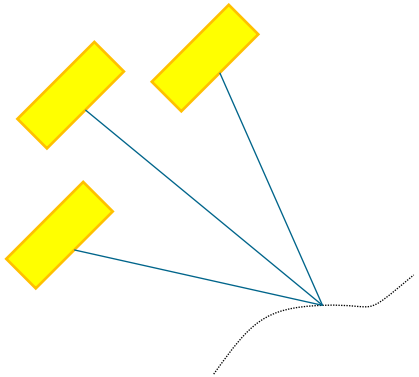
Polarimetry



Frequency



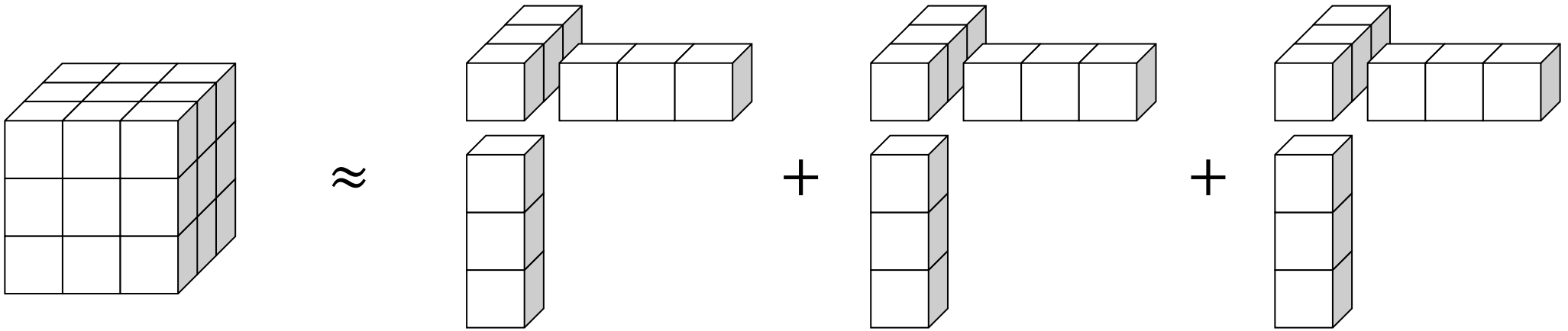
Interferometry / Tomography



Methods to jointly analyze multidimensional SAR data are of interest

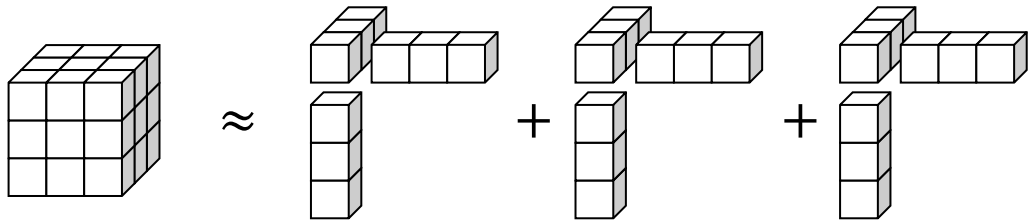
Canonical Polyadic (CP) Decomposition

$$\mathcal{X} \approx \sum_{r=1}^{R} a_r \circ b_r \circ c_r$$



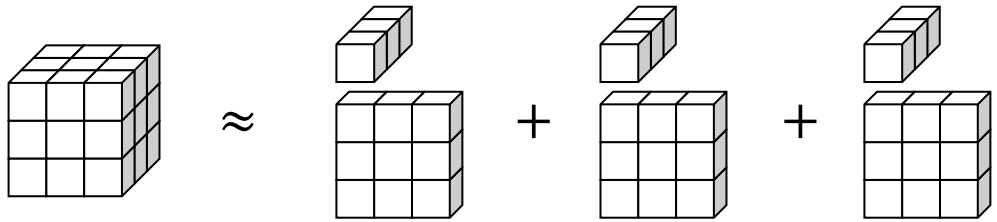
CP decomposition represents the tensor as a sum of components

Extending CP to SAR Data



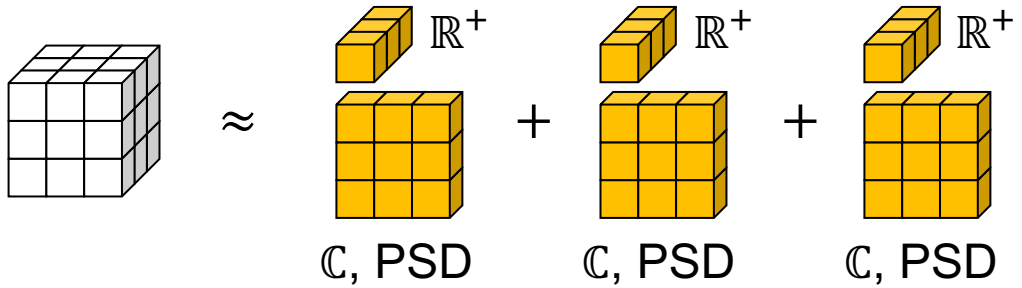
Standard CP decomposition

- Same number set ($\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}^+$) for tensor and factors
- Factors are vectors



Flexible factor shapes

→ Allow matrices (or tensors) as factors

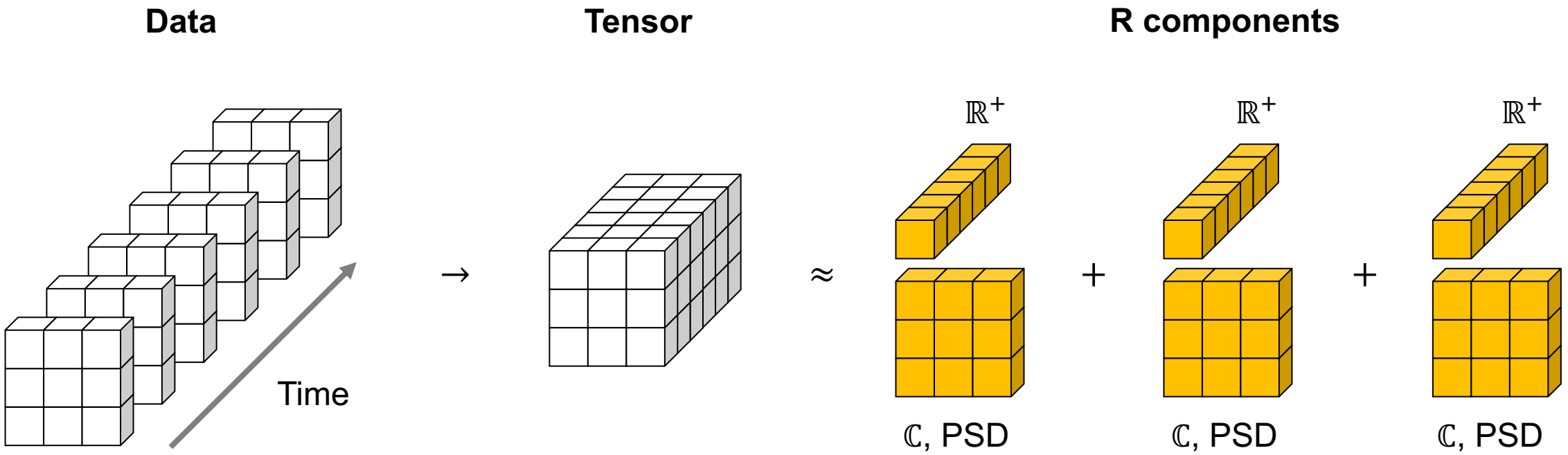


Flexible factor constraints

→ Allow constraints for physical validity

CP decomposition requires extensions in order to preserve the structure of SAR data

Polarimetric Time Series Decomposition



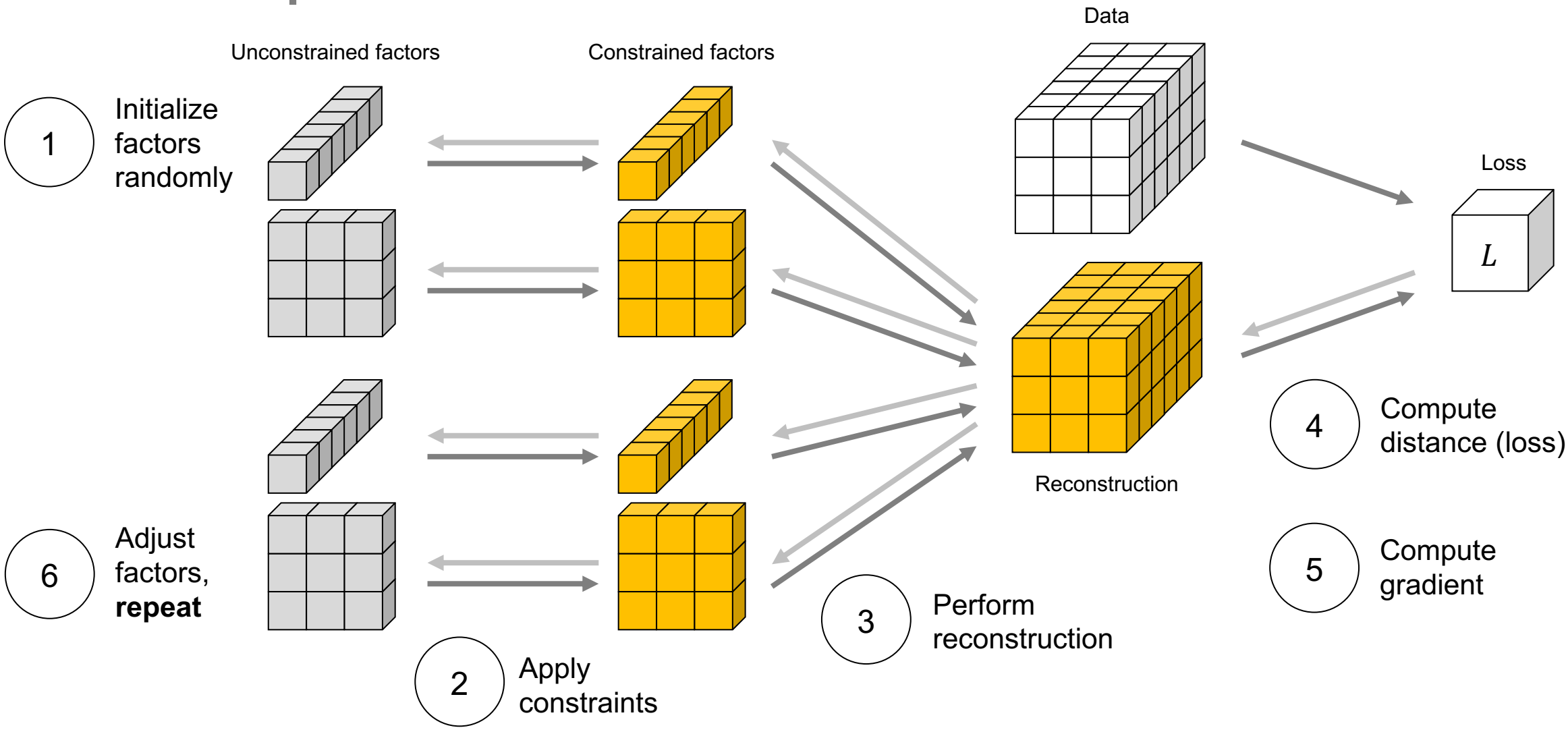
Polarimetric time series with
N coherency matrices

Nx3x3 Tensor

Positive time factors,
Positive semi-definite (PSD)
rank-1 complex polarimetry factors

How to obtain the factors?

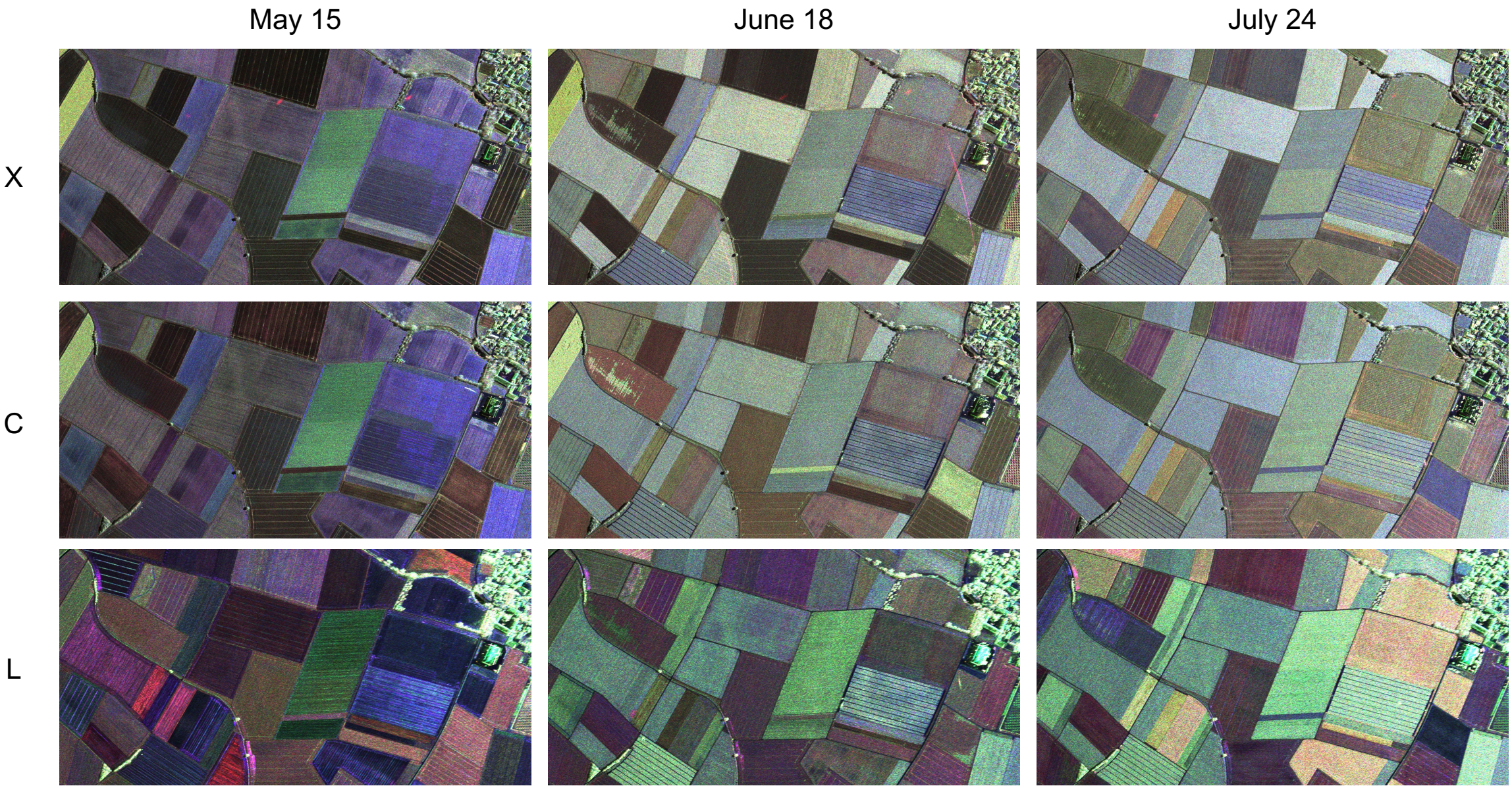
Iterative Optimization



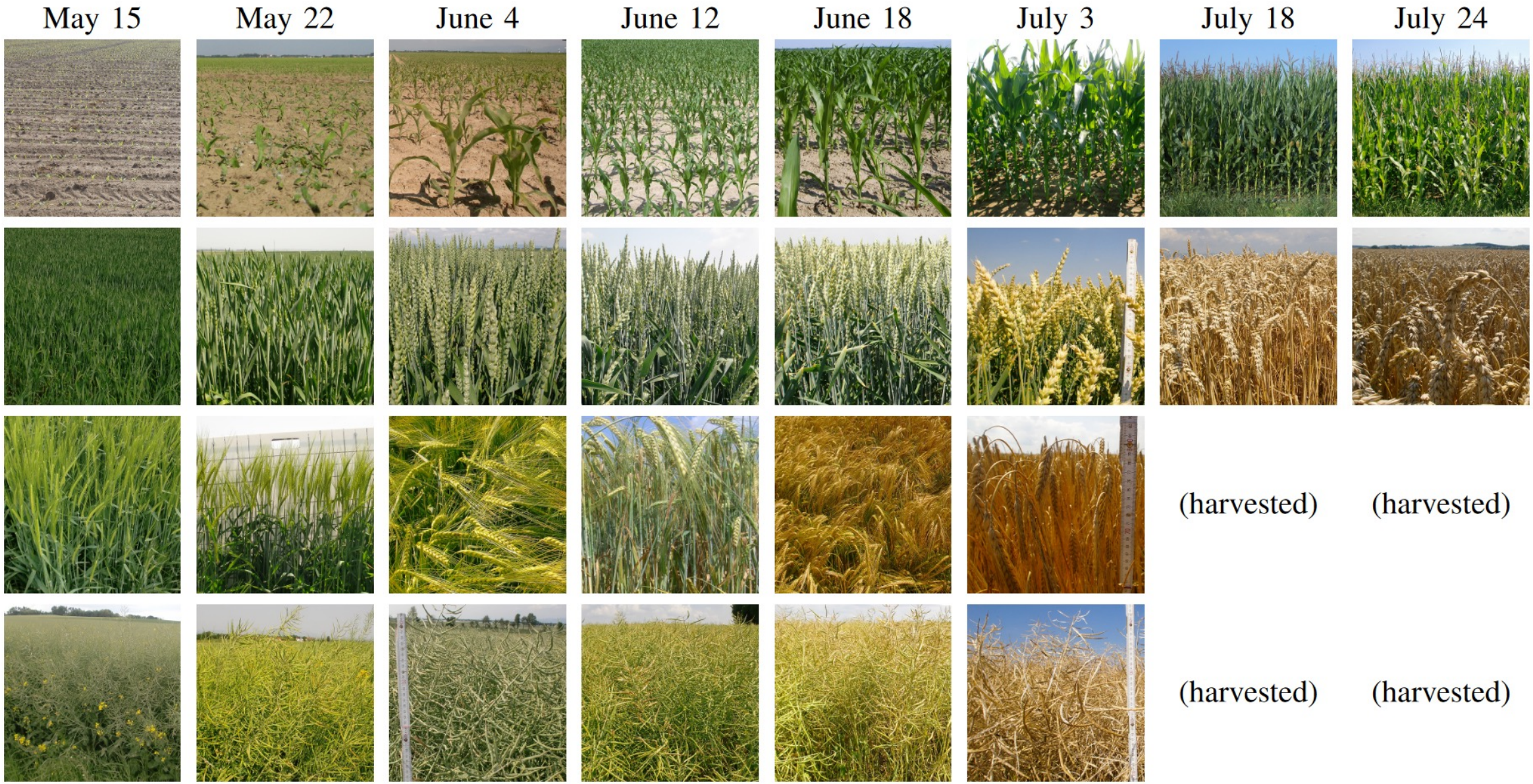
Constrained decomposition factors can be obtained through optimization

POLARIMETRIC CHANGE ANALYSIS

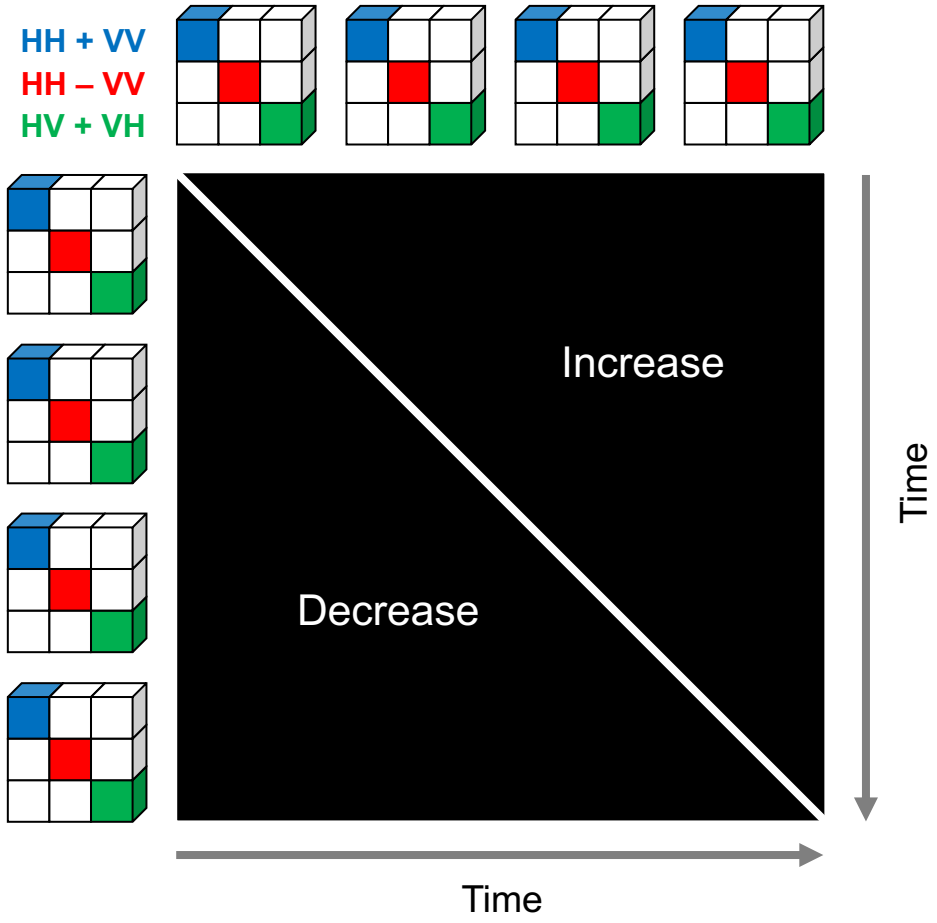
CROPEX 2014 Campaign

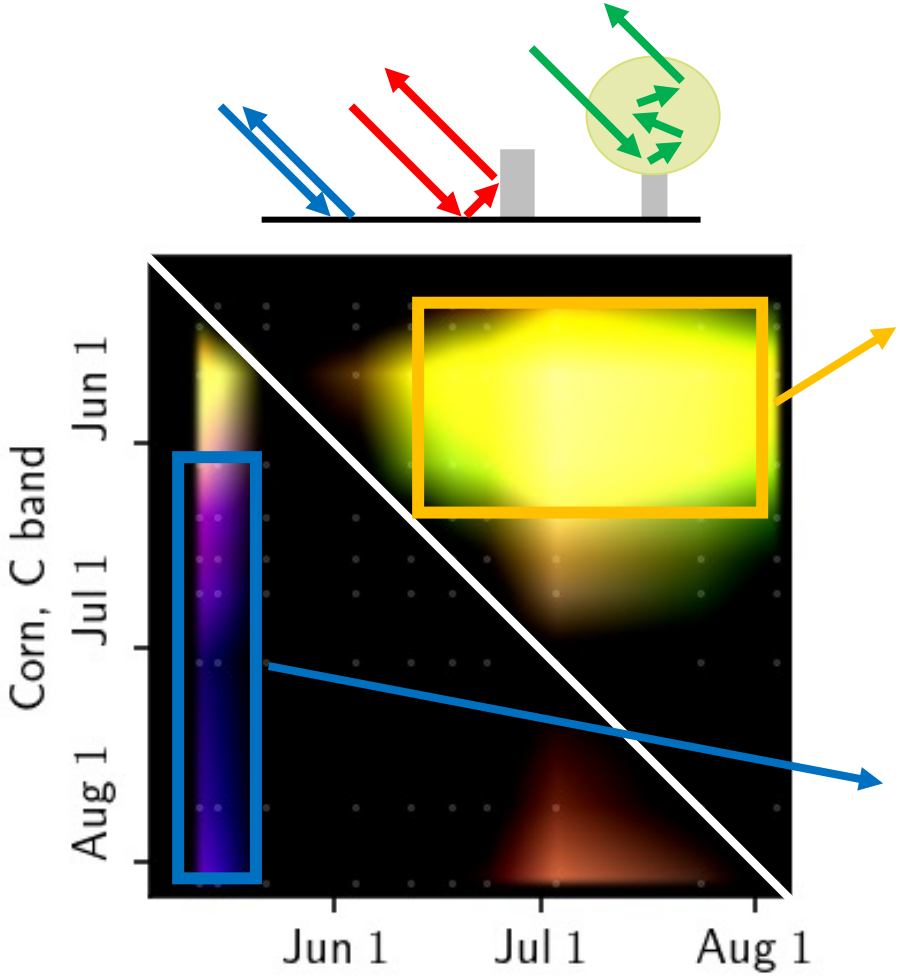


CROPEX 2014 Campaign



Polarimetric Change Analysis



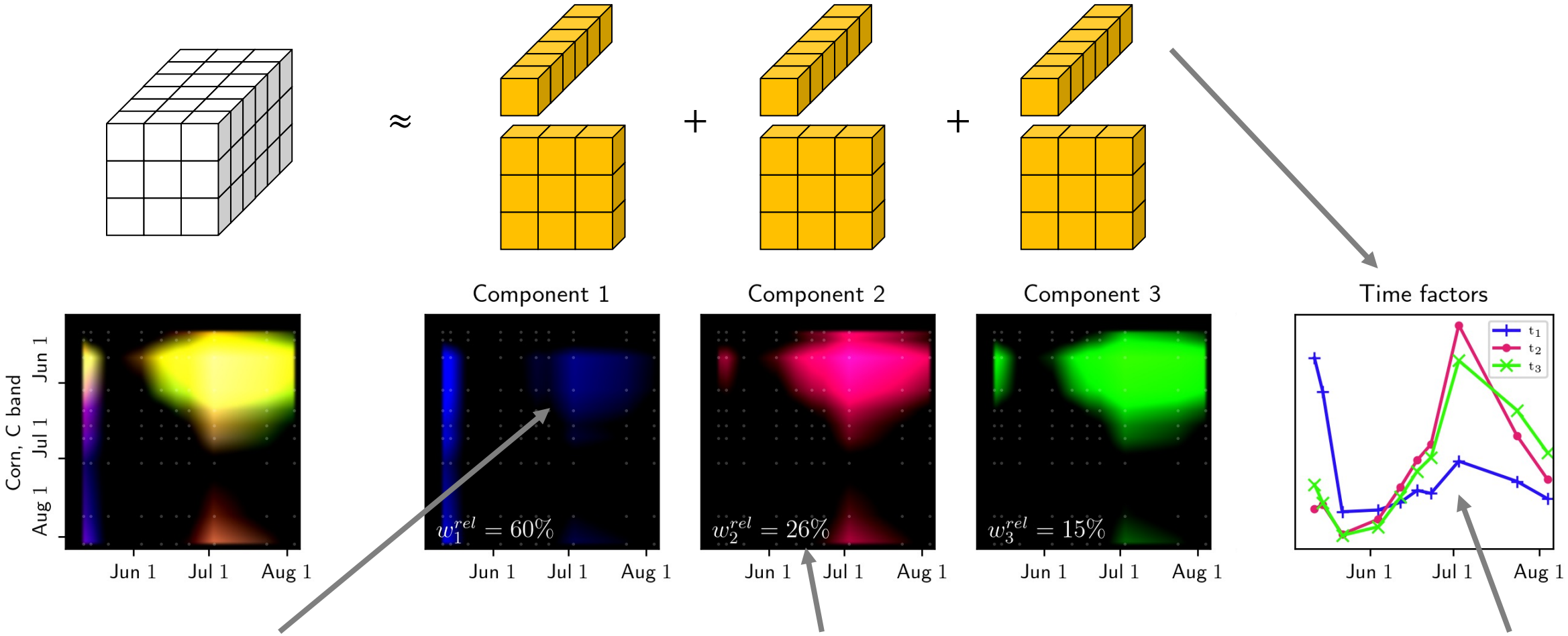


More volume

Less surface

Change matrix represents changes between each pair of acquisitions

More Detailed Change Detection
Corn, C Band

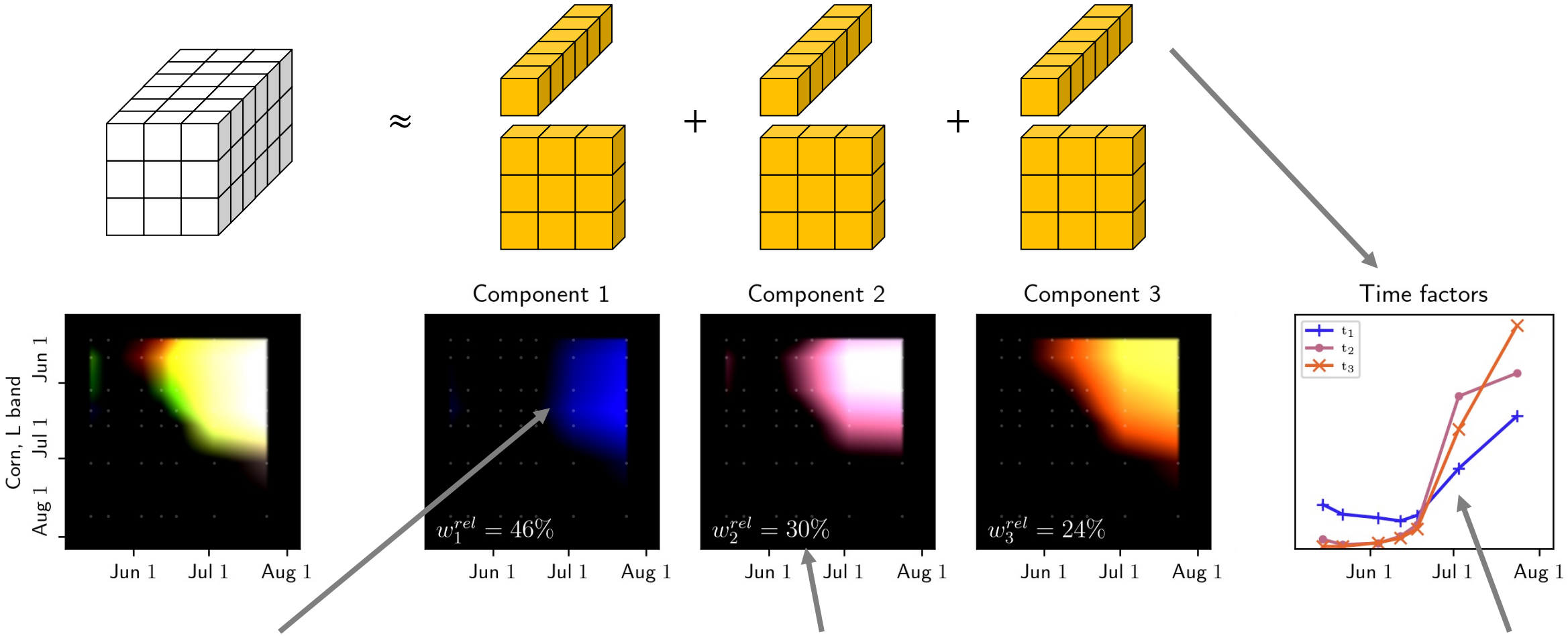


Finer details are visible after the decomposition

Component weights indicate importance

Time factors offer simpler interpretation

More Detailed Change Detection
Corn, L Band



Finer details are visible after the decomposition

Component weights indicate importance

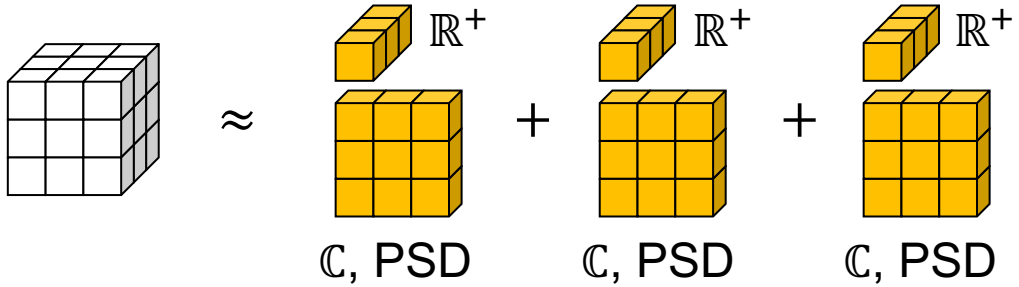
Time factors offer simpler interpretation

EXTENSIONS

Decomposing Different Data Dimensions

Polarimetric Time Series Decomposition

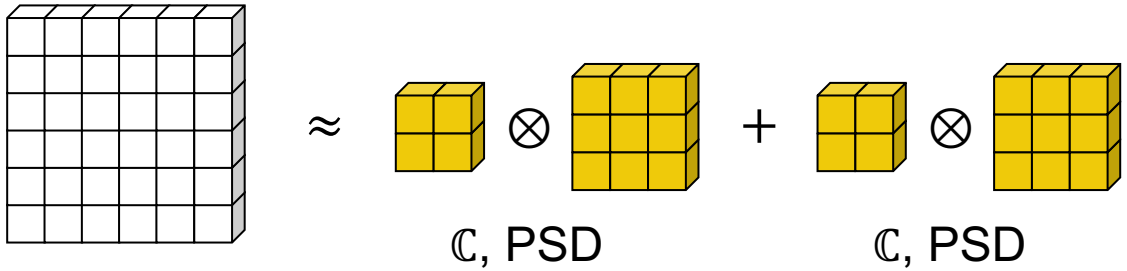
Polarimetric time series



Polarimetry + Time

Sum of Kronecker Products (SKP) Decomposition

Multibaseline multipolarization matrix



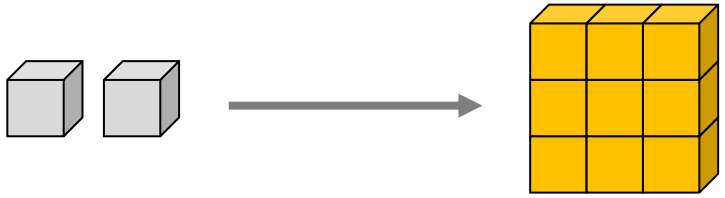
Polarimetry + Structure

Joint analysis and decomposition of different data dimensions possible

Integration of Physical Models

Constraints

Unconstrained factors → Constrained factors



Physical Model

Physical parameters → Constrained factors

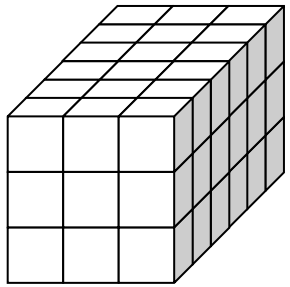
Benefits of Model Integration

- Simpler interpretation: Model-based components have a clear meaning
- Parameter inversion: We obtain physical parameters
- Larger observation space: More complex models possible

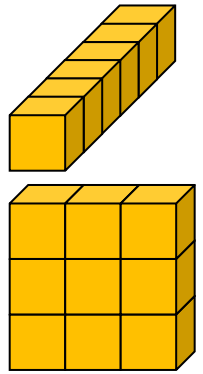
Summary

Constrained Tensor Decomposition Framework

- ✓ Joint decomposition and analysis of different data dimensions
- ✓ Physical validity enforced through constraints
- ✓ Extensible framework obtains solution through optimization

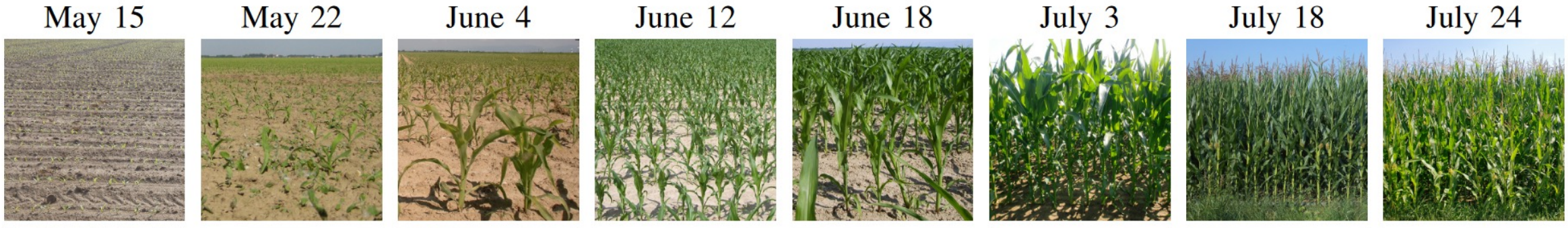


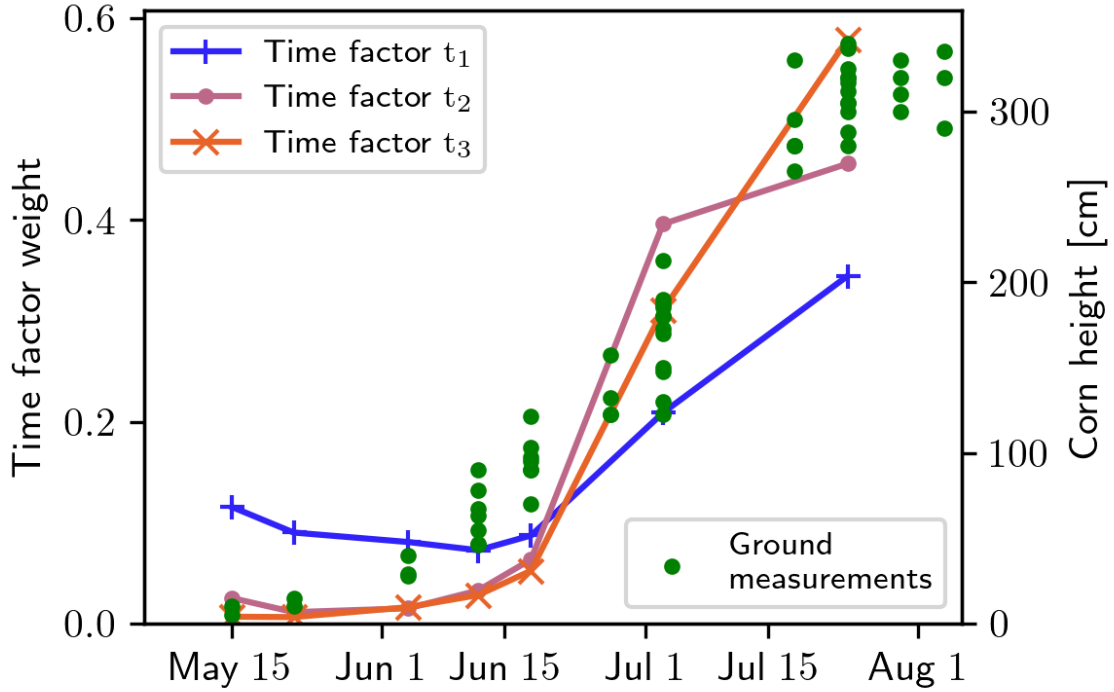
Thank You!
Questions?



BACKUP SLIDES

Time Factor Analysis, Corn, L Band

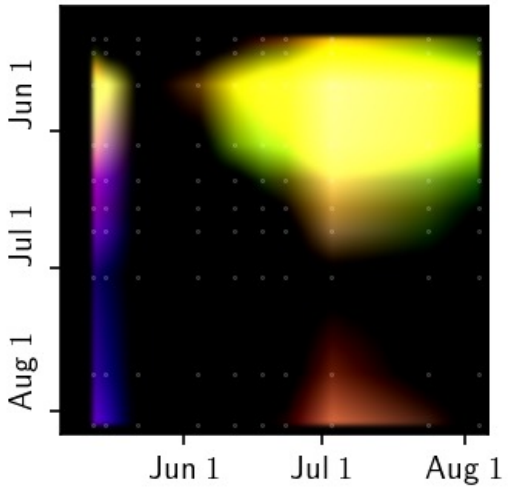




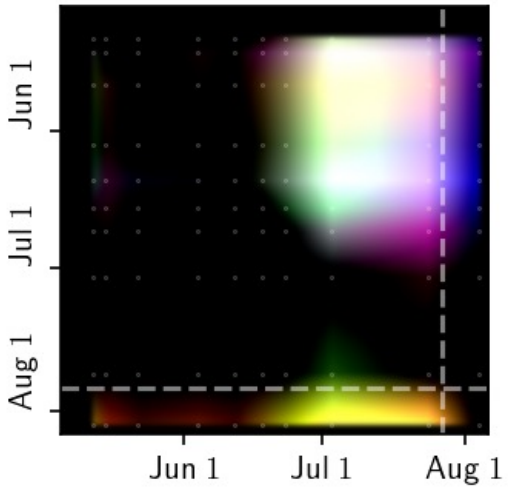
Corn time factors in L band show a correlation to crop height

Change Matrices for Different Crops
C Band

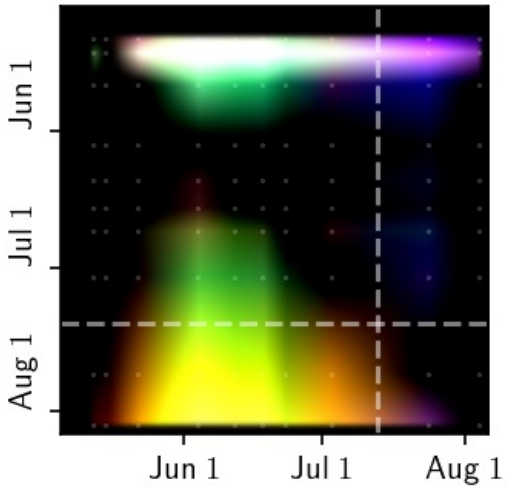
Corn, C Band



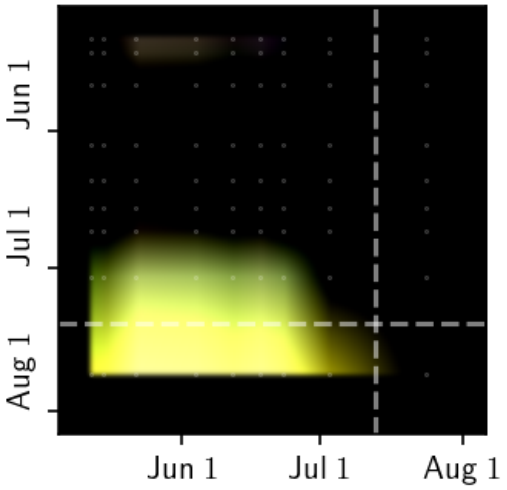
Wheat, C Band



Barley, C Band



Rapeseed, C Band

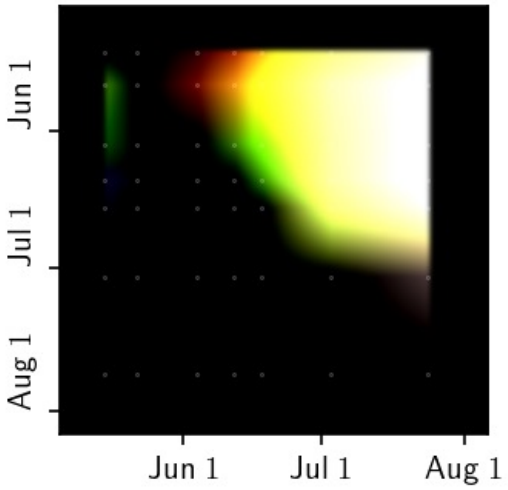


HH + VV
HH – VV
HV + VH

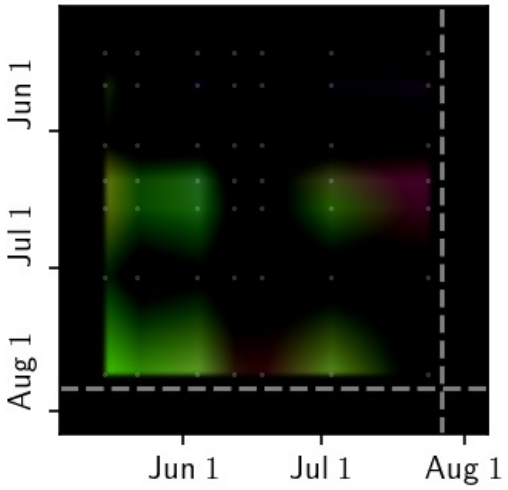
Different crop types show different changes

Change Matrices for Different Crops
L Band

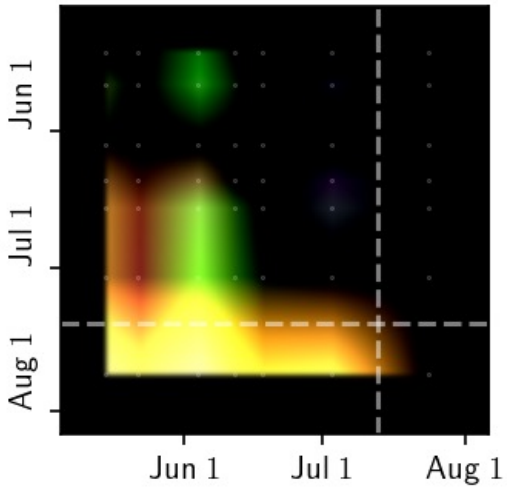
Corn, L Band



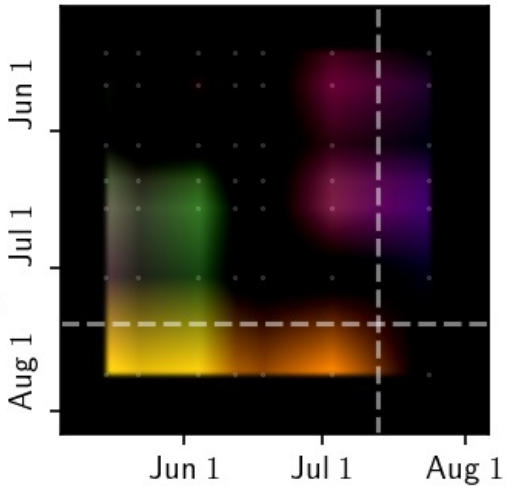
Wheat, L Band



Barley, L Band



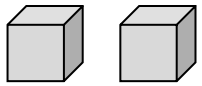
Rapeseed, L Band



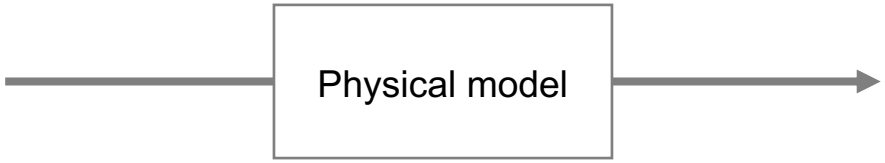
HH + VV
HH – VV
HV + VH

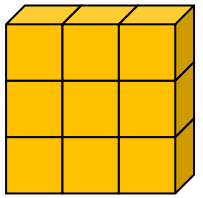
Different radar bands are sensitive to different scales

Physical Models

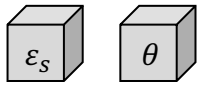


Physical parameters





Predicted backscatter



Dielectrics & Incidence

$$R_h := \frac{\cos\theta - \sqrt{\varepsilon_s - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon_s - \sin^2\theta}}$$

$$R_v := \frac{(\varepsilon_s - 1)\left(\sin^2\theta - \varepsilon_s(1 + \sin^2\theta)\right)}{(\varepsilon_s \cos\theta + \sqrt{\varepsilon_s - \sin^2\theta})^2}$$

$$f_s = \frac{m_s^2}{2}|R_h + R_v|^2 \quad \beta = \frac{R_h - R_v}{R_h + R_v}$$

Bragg surface scattering model

$$f_s \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Predicted backscatter

Physical models predict signal from parameters