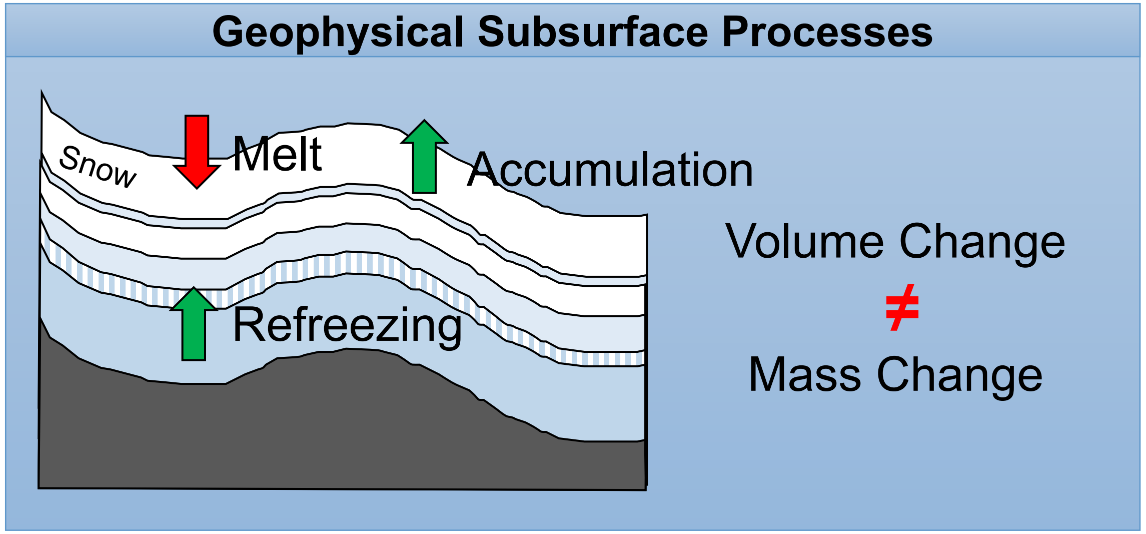
TOWARDS A POL-INSAR FIRN DENSITY RETRIEVAL

Georg Fischer, Matteo Pardini, Kostas Papathanassiou, Irena Hajnsek
(georg.fischer@dlr.de)

DLR

Motivation from Glaciological Perspective



Uncertainties:

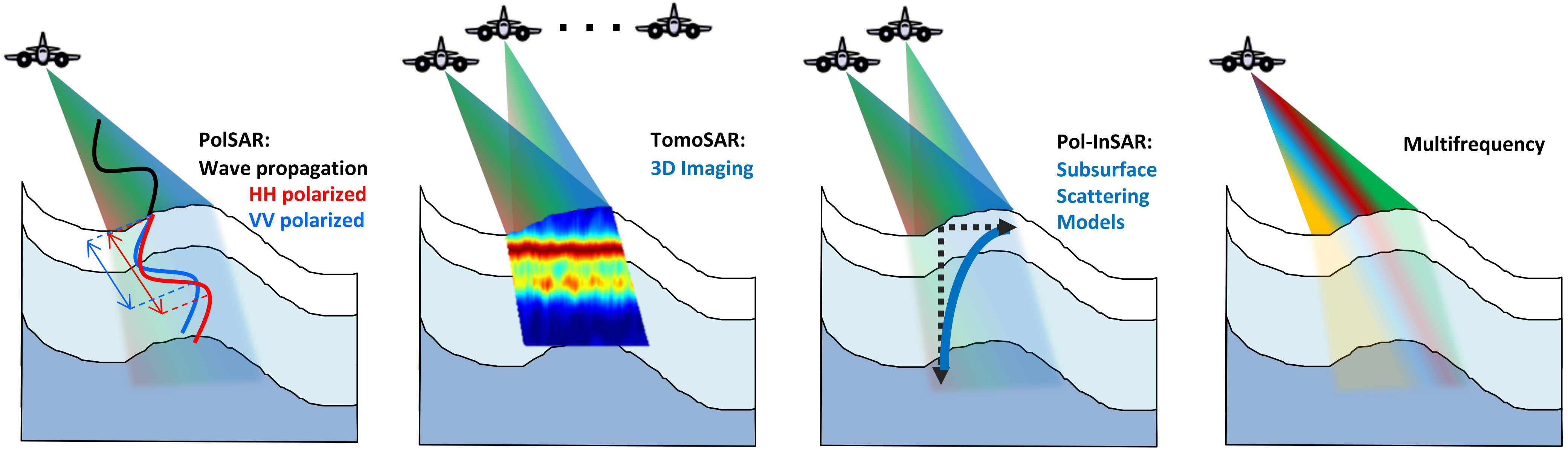
- Density for the volume to mass conversion in mass balance estimations
- Refreezing of melt water and melt water retention in firn
- Horizontal and vertical heterogeneity

[IPCC AR5, 2013; Parry et al., 2007]

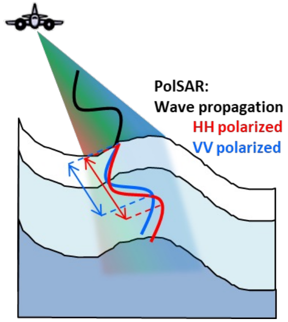
Research Questions from Pol-InSAR Perspective

What is the information content about the subsurface?

- Anisotropic propagation
- Vertical scattering structure
- …and their frequency dependence

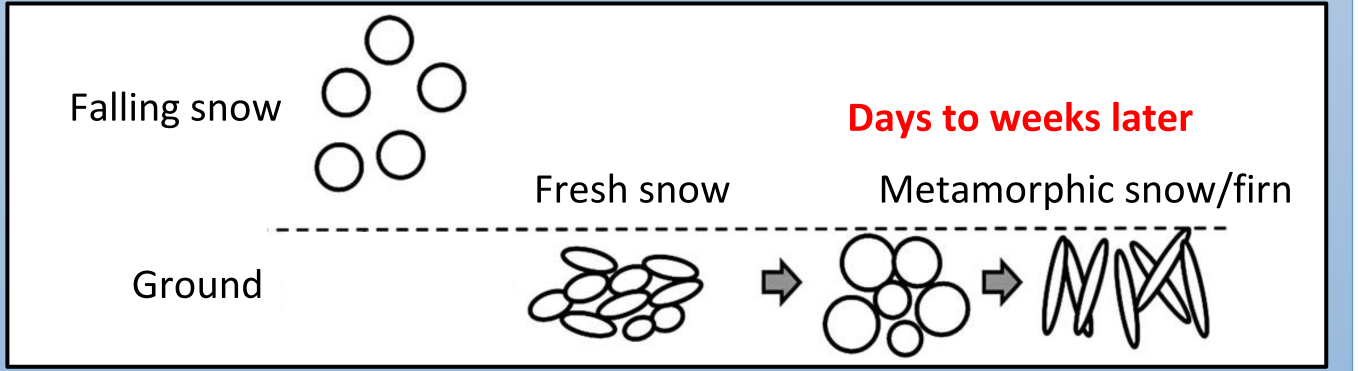


Polarimetry

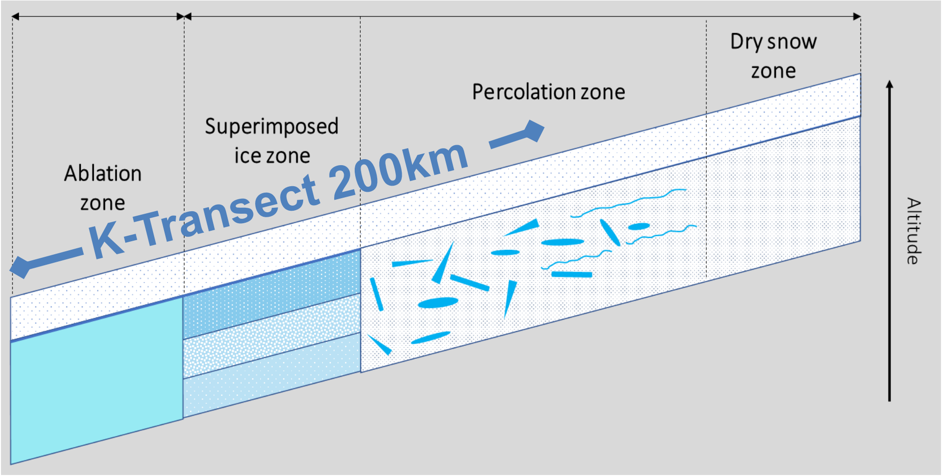


Temperature Gradient Metamorphism

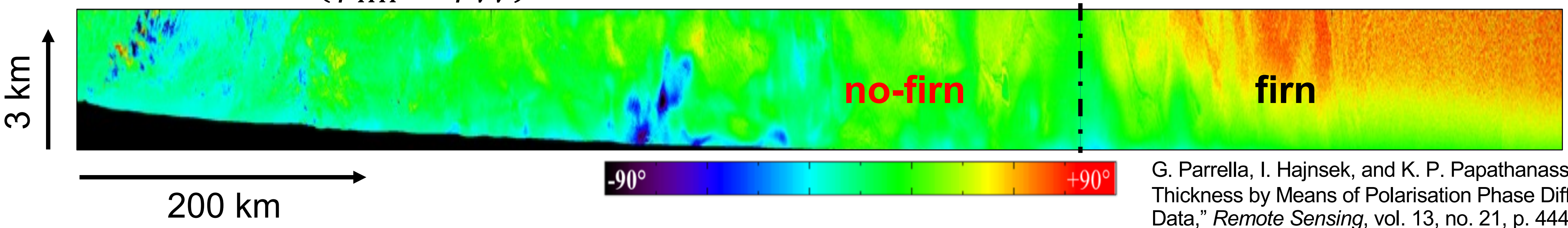
→ Vertical firn grains



Anisotropic propagation in firn → $\mathrm{CPD} = \phi_{HH} - \phi_{VV}$

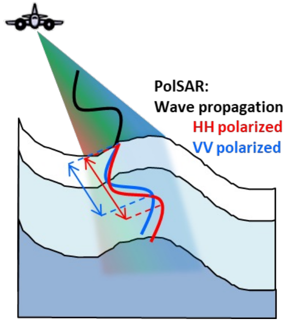


L-band CPD $(\phi_{HH} - \phi_{VV})$



G. Parrella, I. Hajnsek, and K. P. Papathanassiou, “Retrieval of Firn Thickness by Means of Polarisation Phase Differences in L-Band SAR Data,” *Remote Sensing*, vol. 13, no. 21, p. 4448, Nov. 2021,

PolSAR Model: From CPD to Firn Properties



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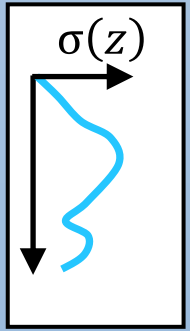
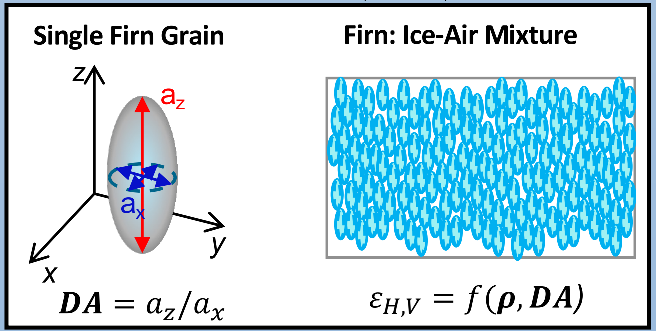
Firn Anisotropy Model

Rationale: Temperature gradient metamorphism
→ Dielectric anisotropy of firn
→ Co-polar Phase Difference (CPD)

$$CPD = \phi_{HH} - \phi_{VV} = f(\boldsymbol{\sigma}(\boldsymbol{z}), \boldsymbol{\rho}, \boldsymbol{DA})$$

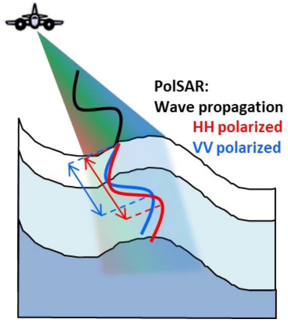
Model parameters:
- Vertical scattering structure $\boldsymbol{\sigma}(\boldsymbol{z})$
- Density $\boldsymbol{\rho}$
- Anisotropy $\boldsymbol{DA}$



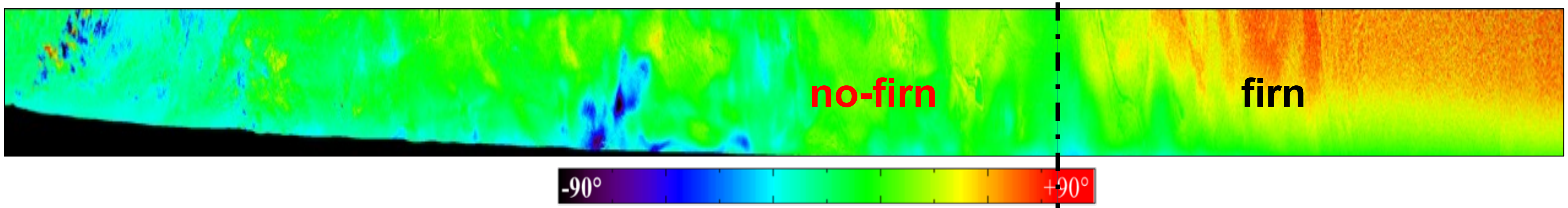


$$CPD = \int \boldsymbol{\sigma}(\boldsymbol{z})\, e^{2\left(-j\frac{2\pi}{\lambda_0}\sqrt{\boldsymbol{\varepsilon_H}}\frac{z}{\cos\vartheta_r}\right)} \left(e^{2\left(-j\frac{2\pi}{\lambda_0}\sqrt{\boldsymbol{\varepsilon_V}}\frac{z}{\cos\vartheta_r}\right)}\right)^{*} dz$$

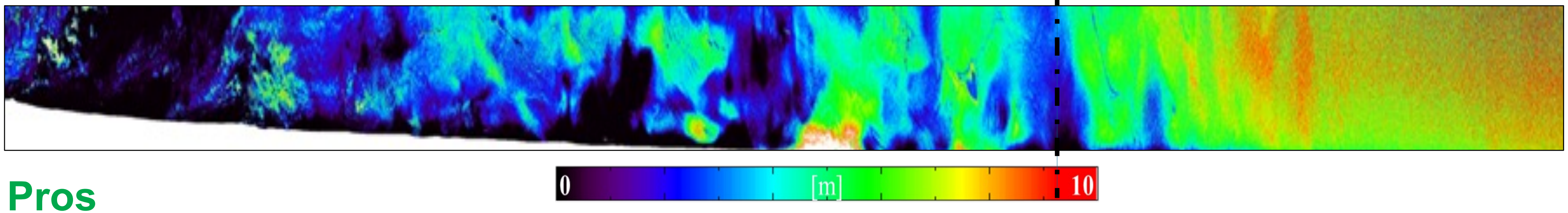
Polarimetry



L-band CPD $(\phi_{HH} - \phi_{VV})$



PolSAR Model: Firn Thickness Estimation



Pros

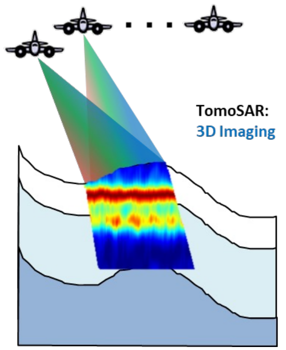
- Direct physical link to density
- Potential to retrieve bulk values from CPD

Cons

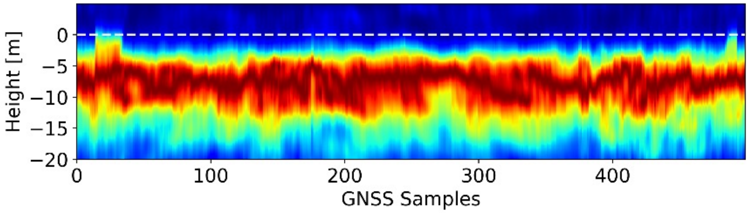
- A-priori knowledge or assumptions necessary
- Model is a strong simplification

G. Parrella, I. Hajnsek, and K. P. Papathanassiou, “Retrieval of Firn Thickness by Means of Polarisation Phase Differences in L-Band SAR Data,” *Remote Sensing*, vol. 13, no. 21, p. 4448, Nov. 2021,

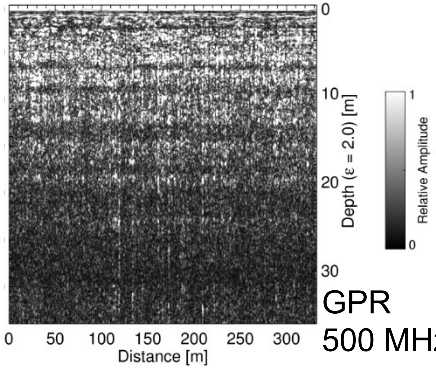
Tomography (P-band Examples)



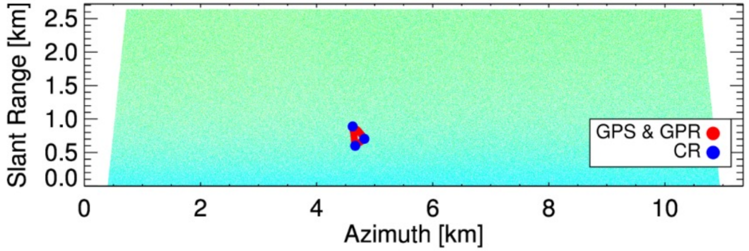
EGIG T05 / Lower Percolation

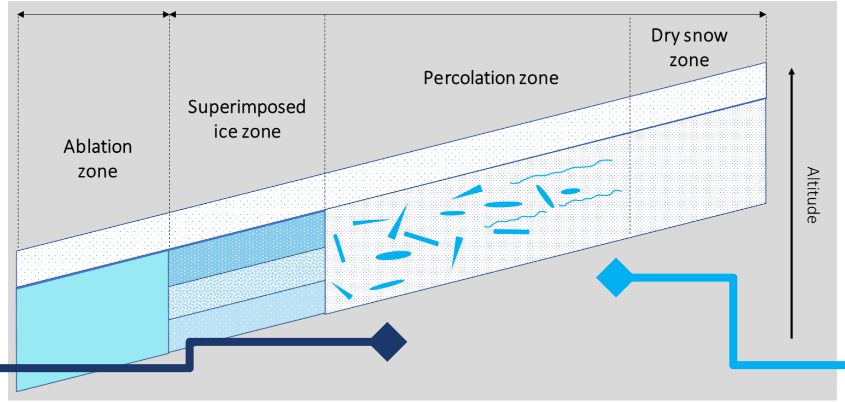


More melting
Uniform scattering
Higher density

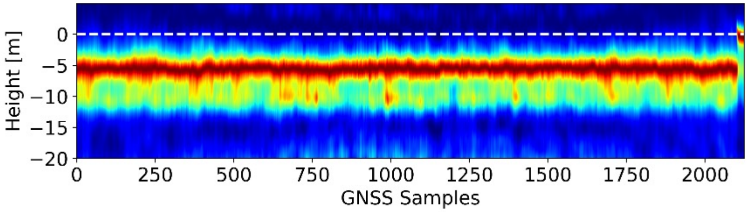


GPR
500 MHz

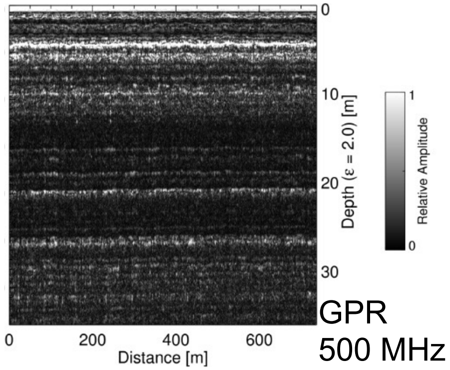




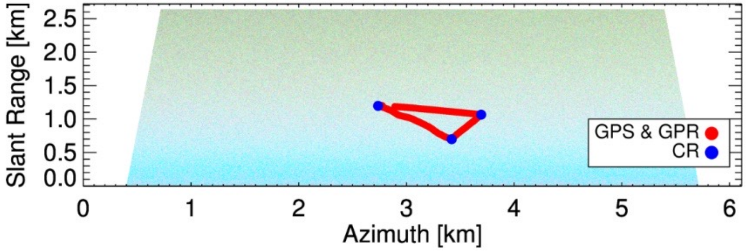
South Dome / Upper Percolation



Less melting
Distinct layers
Lower density



GPR
500 MHz



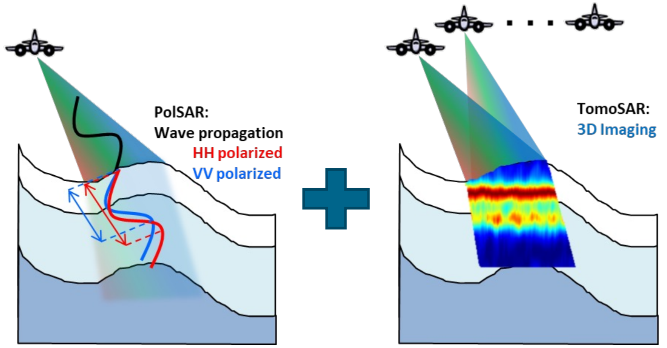
Pros

- Tomography of ice

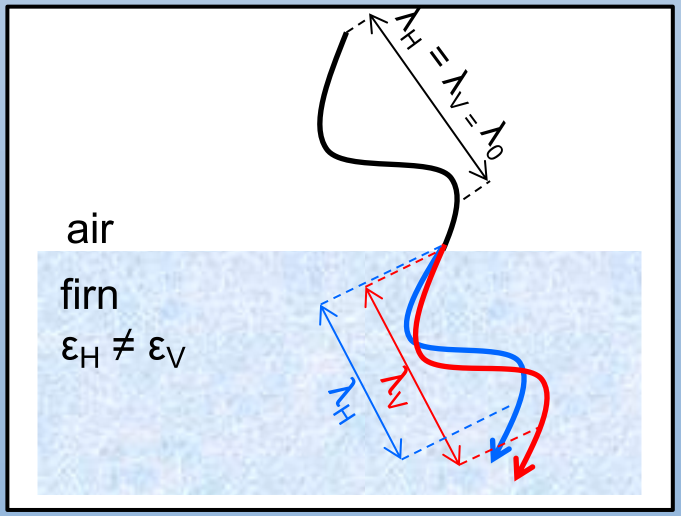
Cons

- Geophysical information needs models
- Observation space

Combination PolSAR + TomoSAR



PolSAR CPD Model

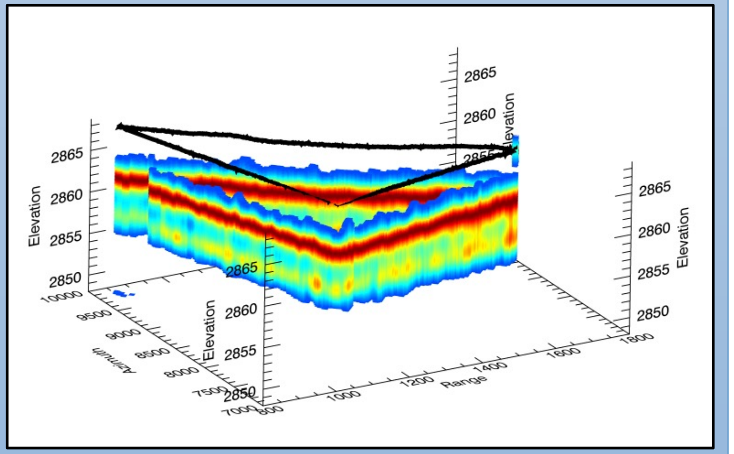


$$CPD = \phi_{HH} - \phi_{VV}$$
$$= f\,(\boldsymbol{\sigma(z)}, \boldsymbol{DA}, \boldsymbol{\rho})$$

Multiple incidence angles

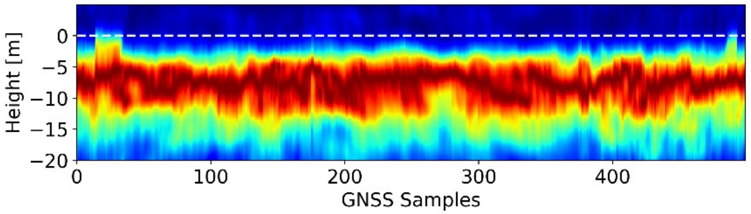
Solving for a bulk density $\boldsymbol{\rho}$

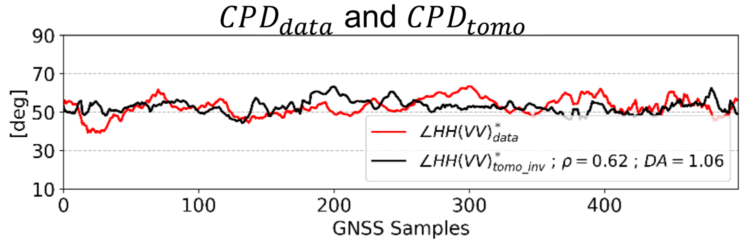
TomoSAR Scattering Structure



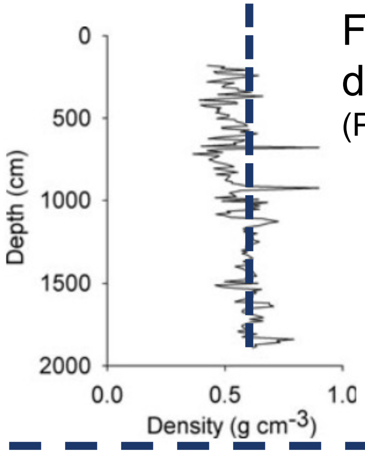
Combination (P-band Results)

EGIG T05

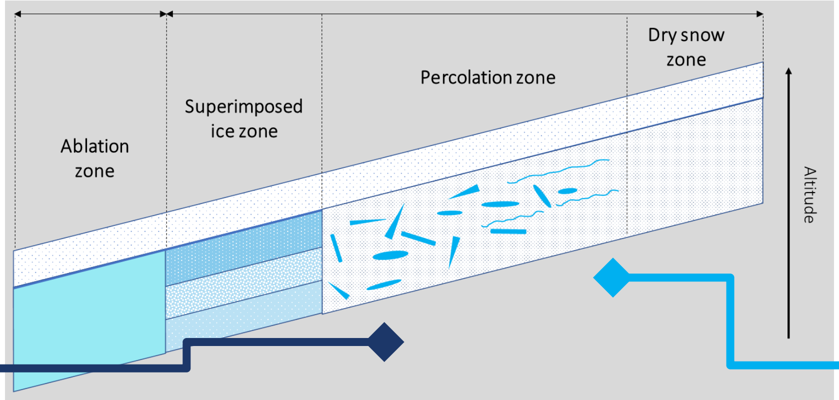




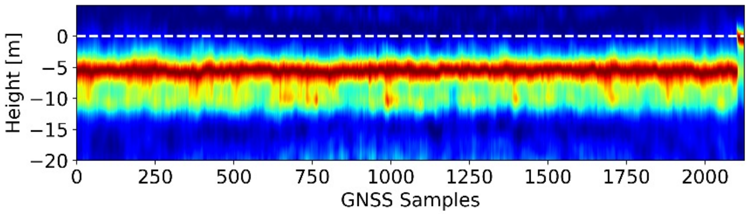
$$\rho = 0.62\ g/cm^3$$
$$DA = 1.06$$

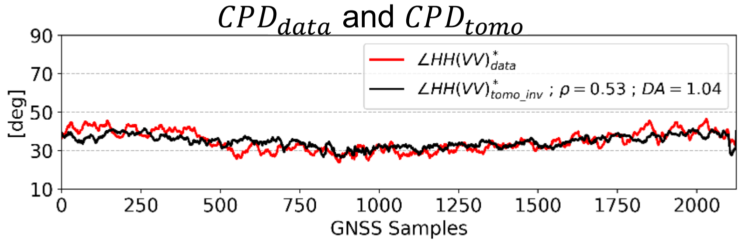


Firn core density (Parry et al., 2007)

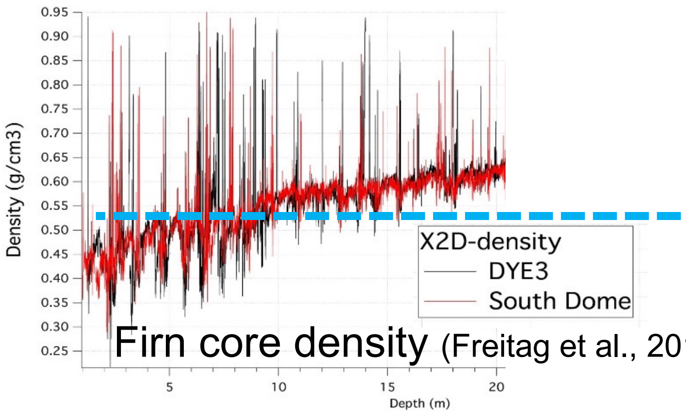


South Dome





$$\rho = 0.53\ g/cm^3$$
$$DA = 1.04$$

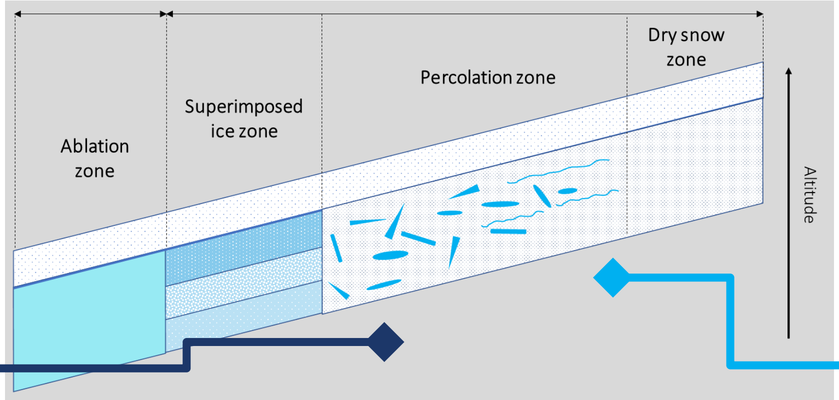


Firn core density (Freitag et al., 2014)

V. Parry *et al.*, “Investigations of meltwater refreezing and density variations in the snowpack and firn within the percolation zone of the Greenland ice sheet,” *Annals of Glaciology*, vol. 46, pp. 61–68, 2007.

J. Freitag, S. Kipfstuhl, S. Hoerz, L. Eling, B. Vinther, and T. Popp, “Melt layer statistic of two firn cores recently drilled at Dye3 and South Dome in the dry snow zone of Southern Greenland,” *presented at the EGU General Assembly*, Vienna, Austria, Apr./May 2014.

Combination (P-band Results)



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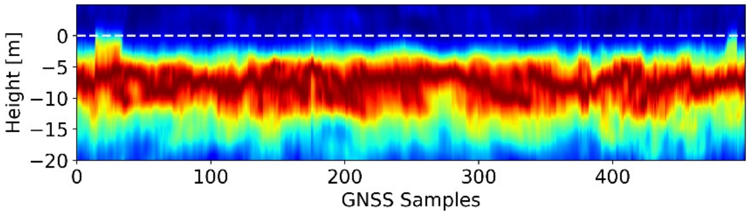

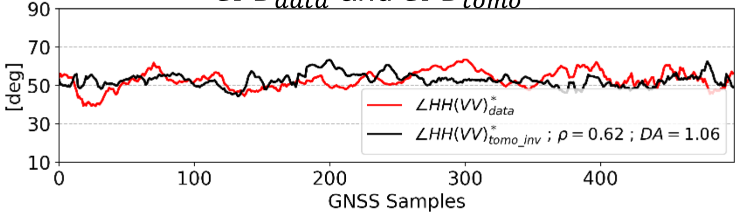

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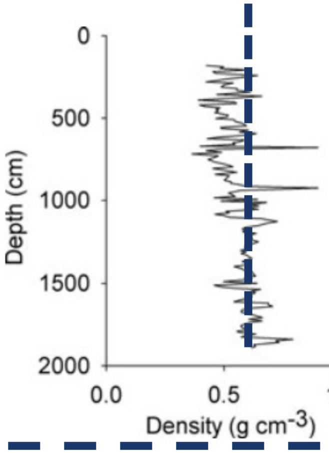


Firn core density (Parry et al., 2007)

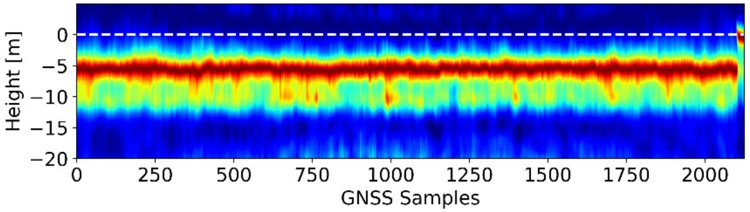
Pros

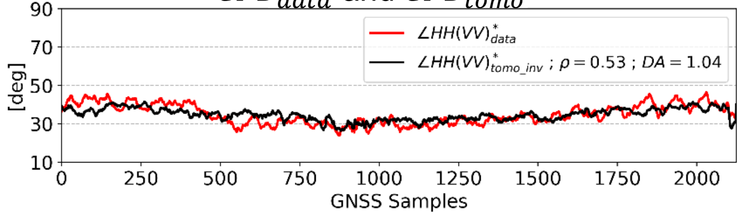
- First density retrieval promising

Cons

- Large observation space used (baselines, incidence angles)
- Sensitivity analysis missing

South Dome





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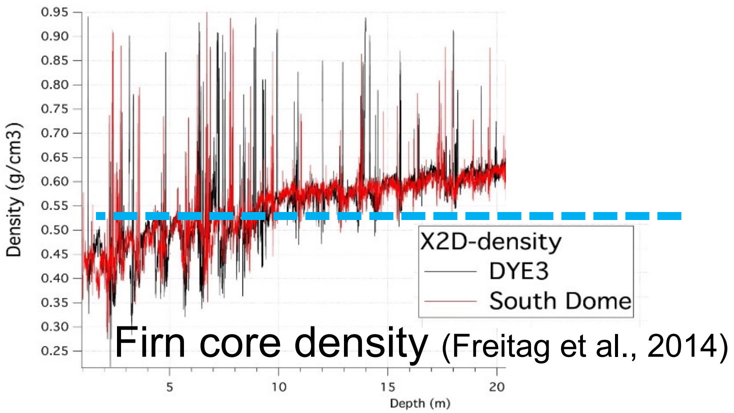

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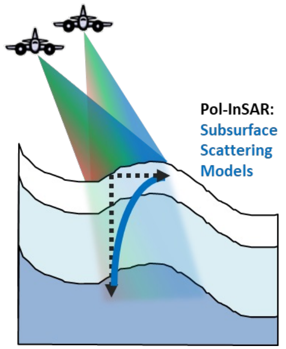
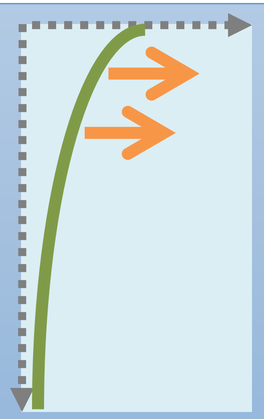
Pol-InSAR Model Inversion



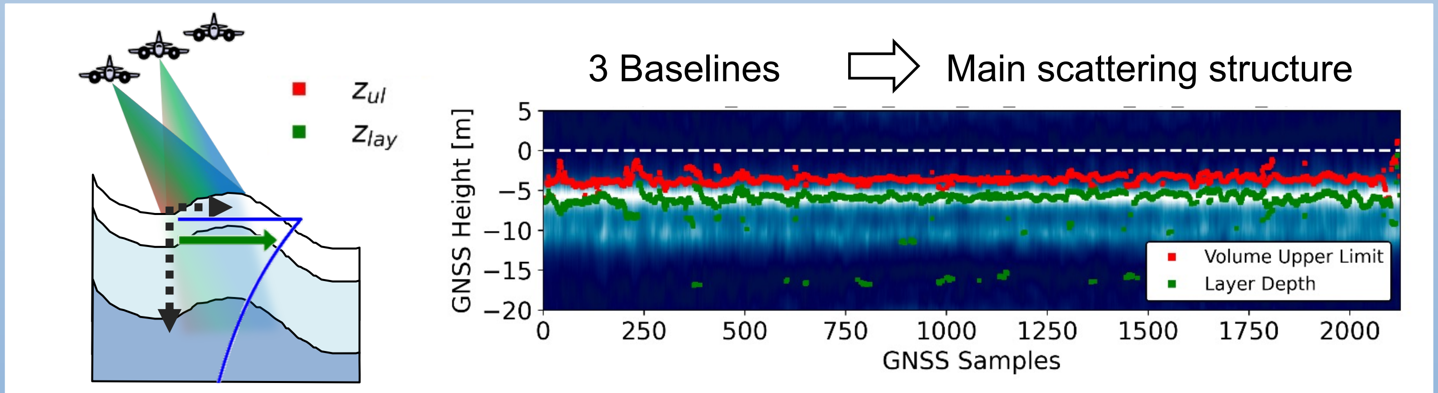
$$\gamma = e^{ik_z z_0} \frac{\gamma_{Vol}(d_{pen}, z_{ul}, \vec{w}) + \sum_{j=1}^{N} m_j(\vec{w}) e^{ik_{zVol} z_j}}{1 + \sum_{j=1}^{N} m_j(\vec{w})}$$

$m_j(\vec{w})$: layer-to-volume ratio
z_j: layer position
N: number of layers
d_{pen}: one-way penetration depth
z_{ul}: upper limit of volume



- Refrozen melt layers → Dirac deltas
- Volume structure function → Uniform Volume with vertical shift

Uniform Volume + 1 Layer Inversion



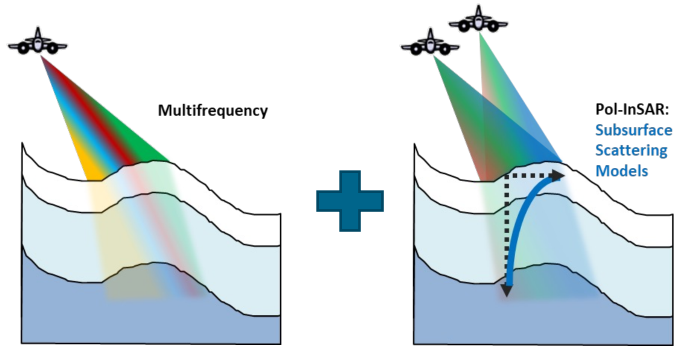
Pros

- Subsurface structure with limited baselines

Cons

- Tradeoff model complexity <> observation space
- Appropriate model setup?

Multifrequency Pol-InSAR



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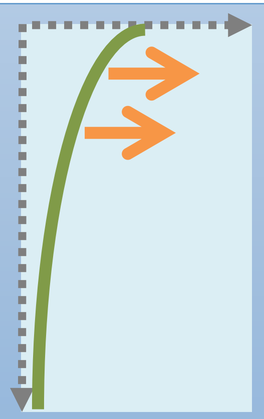
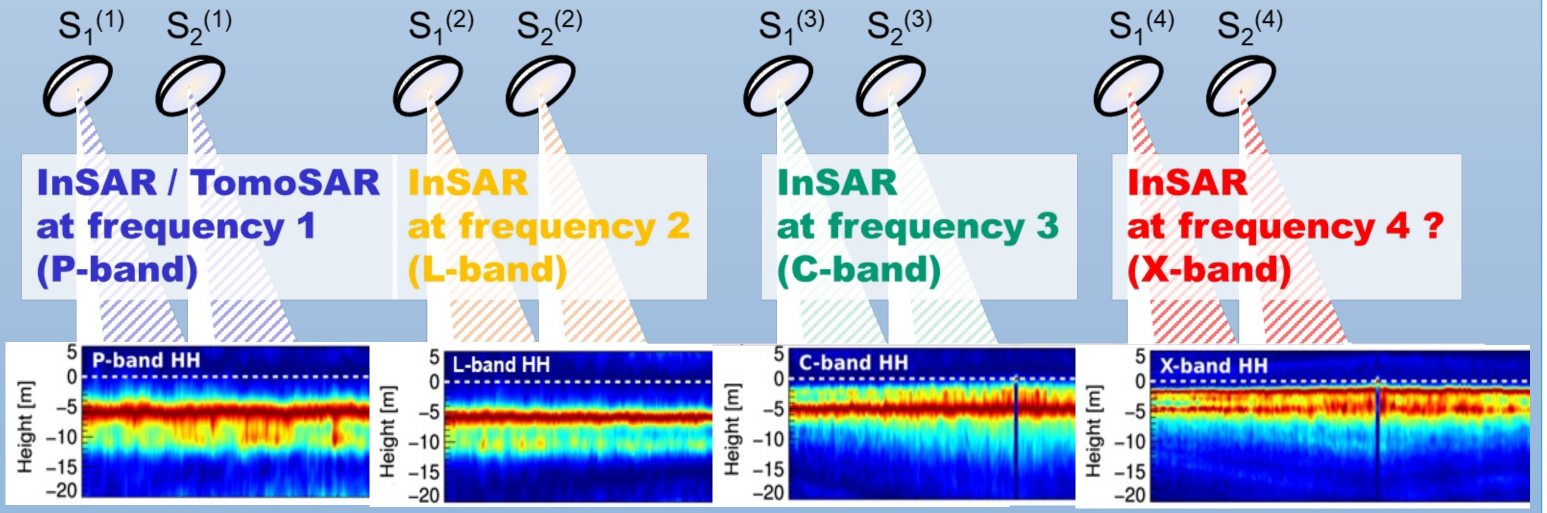


- Refrozen melt layers → Dirac deltas
- Volume structure function → Uniform Volume with vertical shift

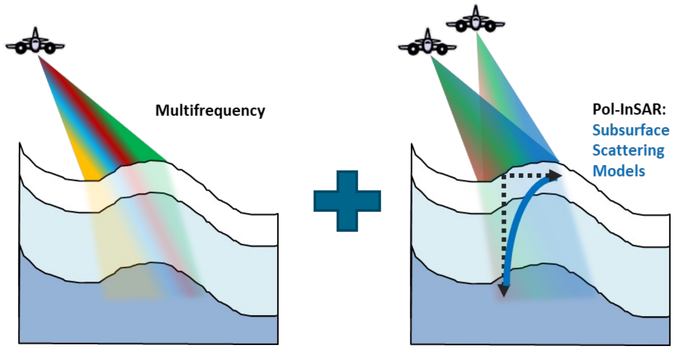
Multifrequency Model Inversion

- Combined detection of layers
- MUSIC estimator
- Limited baselines (Pol-)InSAR

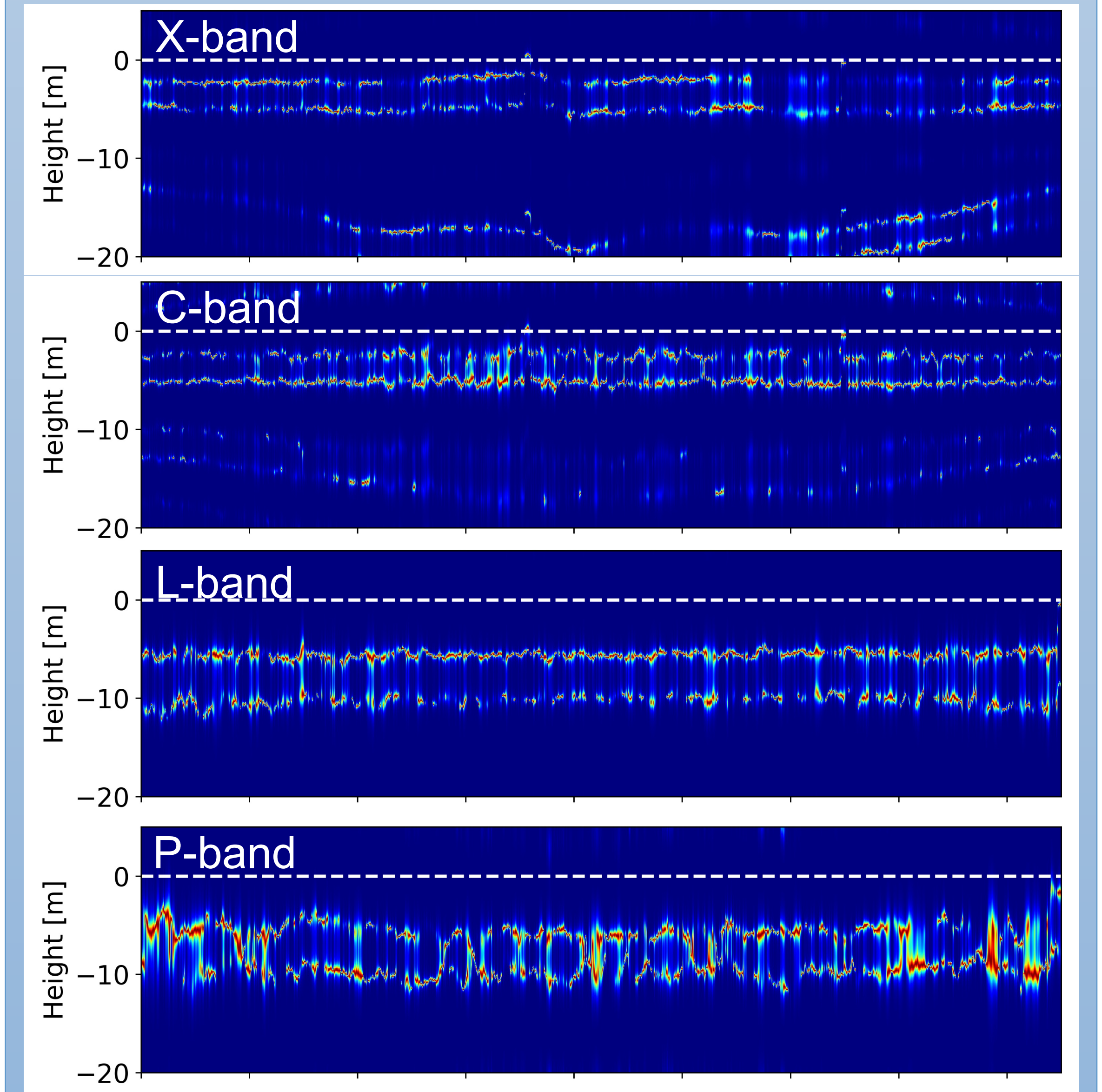
→ More complete 3-D structure information!



Multifrequency MUSIC Layer Detection

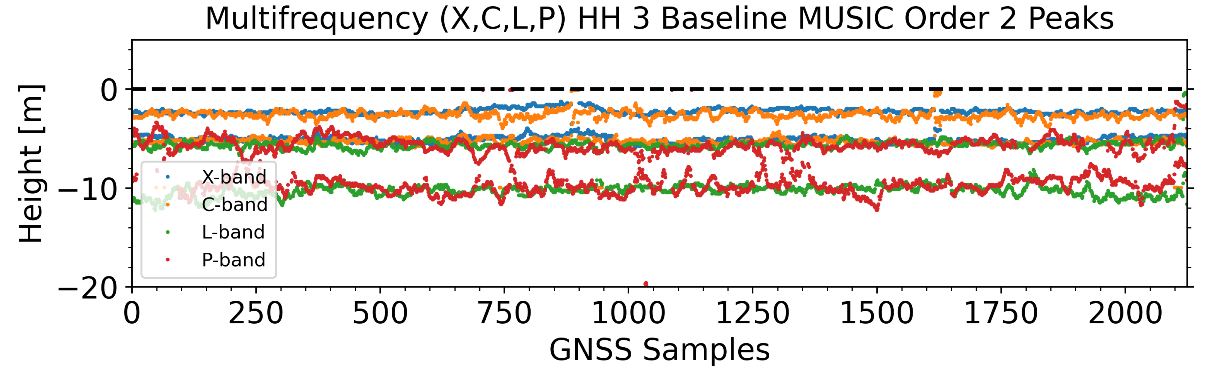


Triple Baseline MUSIC



Layer Detection

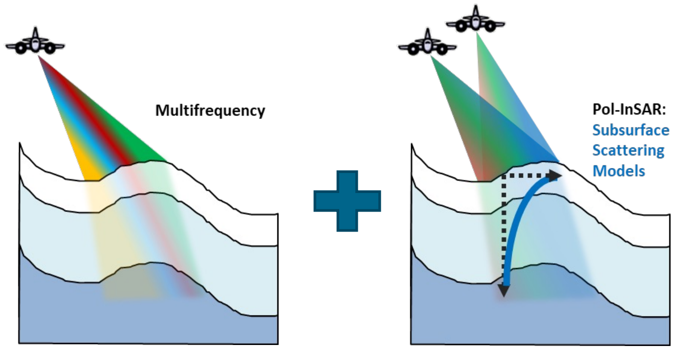
- Order 2 MUSIC / 3 tracks



Results

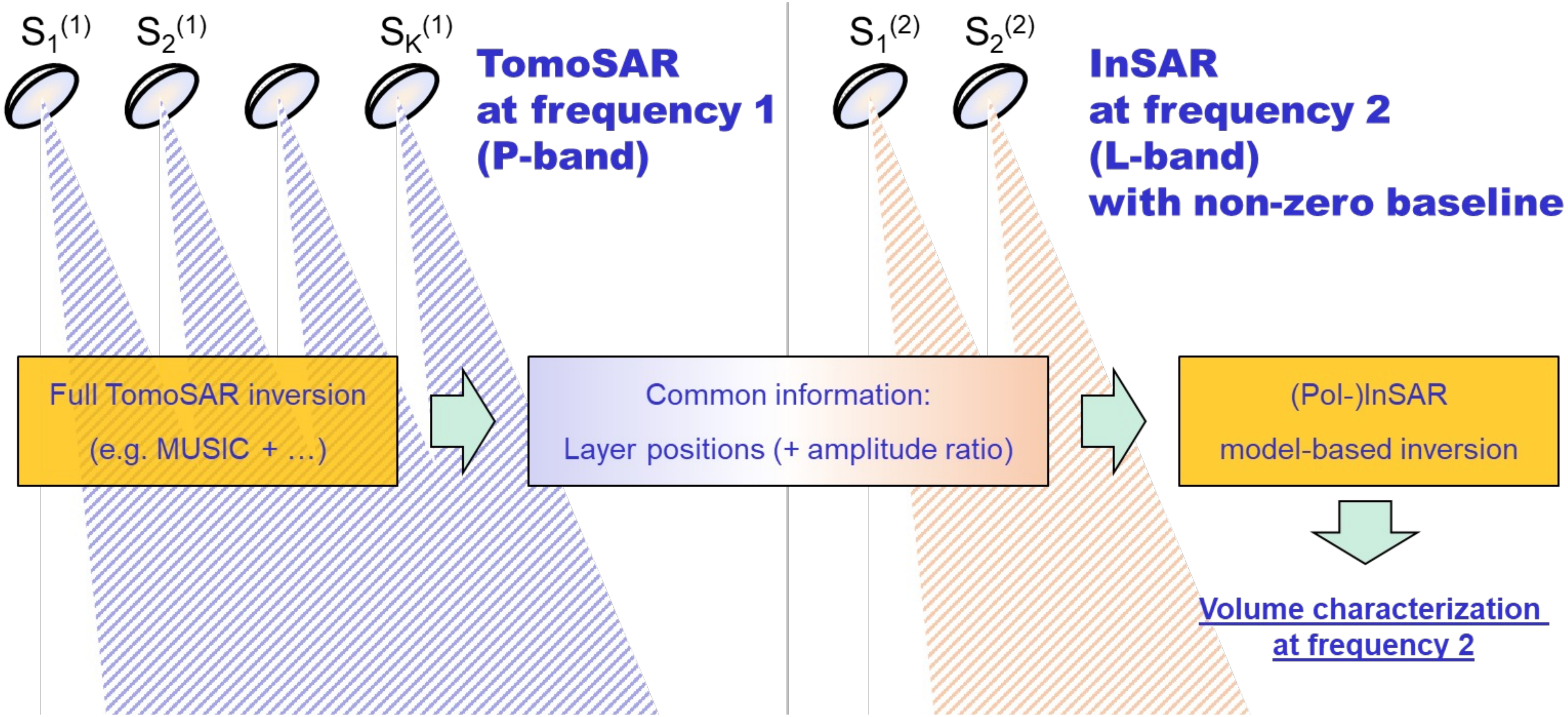
- Good consistency at -2 m, -5 m and -10 m
- Validation with “in situ” layer depths: [-2, -3, -5, -10]

Idea: Transfer Layer Positions across Frequencies

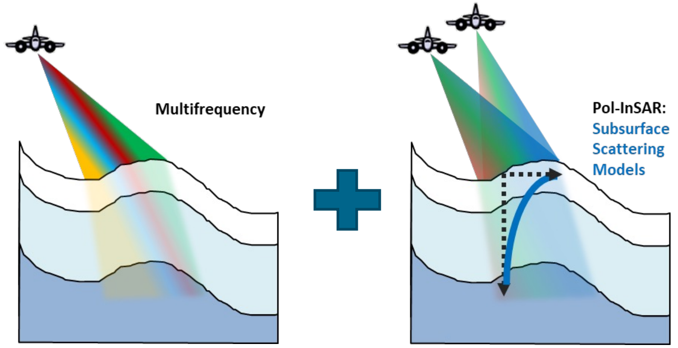


- Layer positions from P-band Tomography to support the model inversion at L-band

→ (Pol-)InSAR inversion of higher complexity at second frequency



L-band 2-layer + UV inversion with P-band MUSIC layer input



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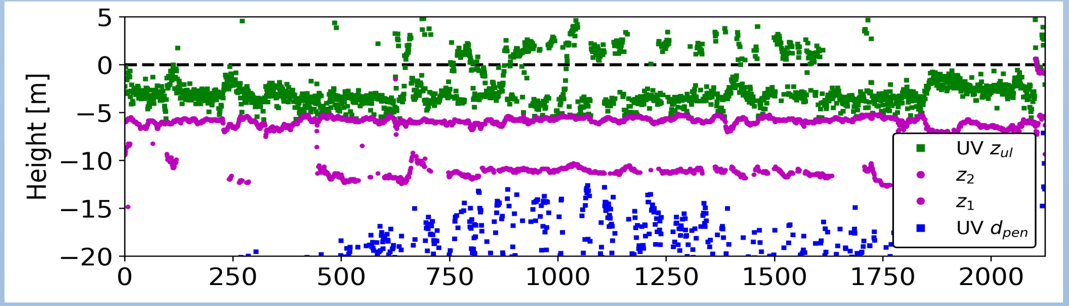
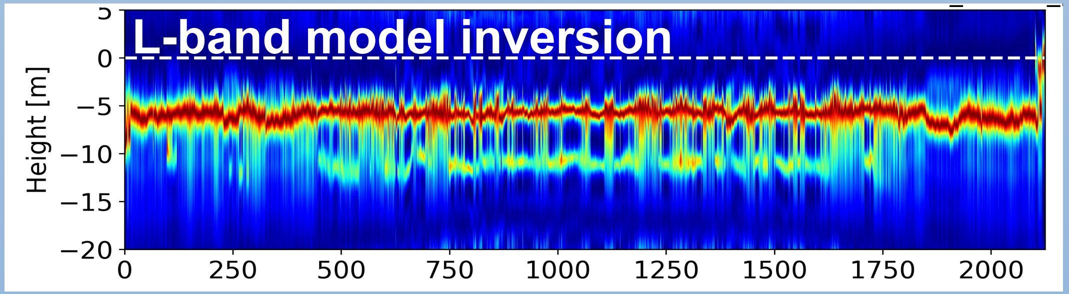
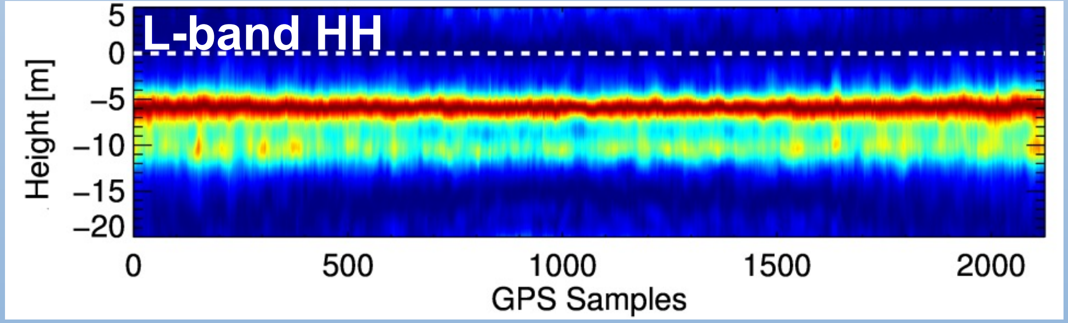
3 Baseline model inversion at **L-band**

- Layer positions z_1, z_2 from P-band MUSIC
- Inversion: $z_{ul}, d_{pen}, m_1, m_2$





Validation: L-band full baseline Tomography



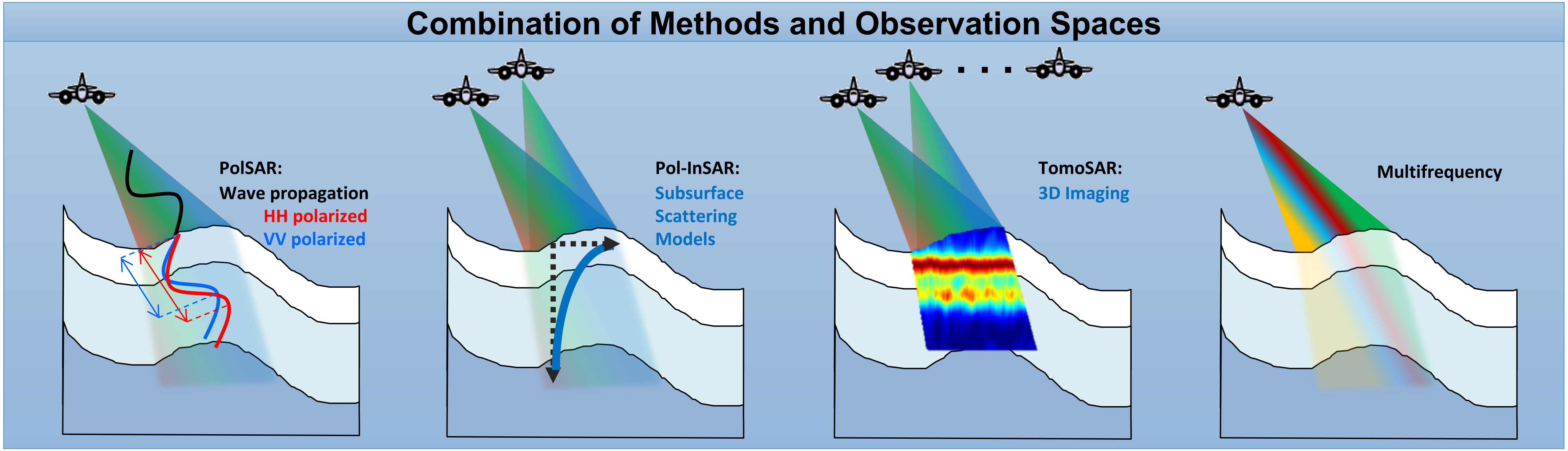
Pros

- Clear additional information content from multifrequency data
- Complementary & common structures

Cons

- Limited availability
- Appropriate exploitation is an open question

Towards Subsurface Information Retrieval



Pros	Anisotropic propagation Model link to density	Subsurface structure model inversion	Subsurface 3D structure	Added information content
Cons	Bulk density, strong approximation	Model complexity tradeoff	Observation space	How to combine and exploit?

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(georg.fischer@dlr.de)

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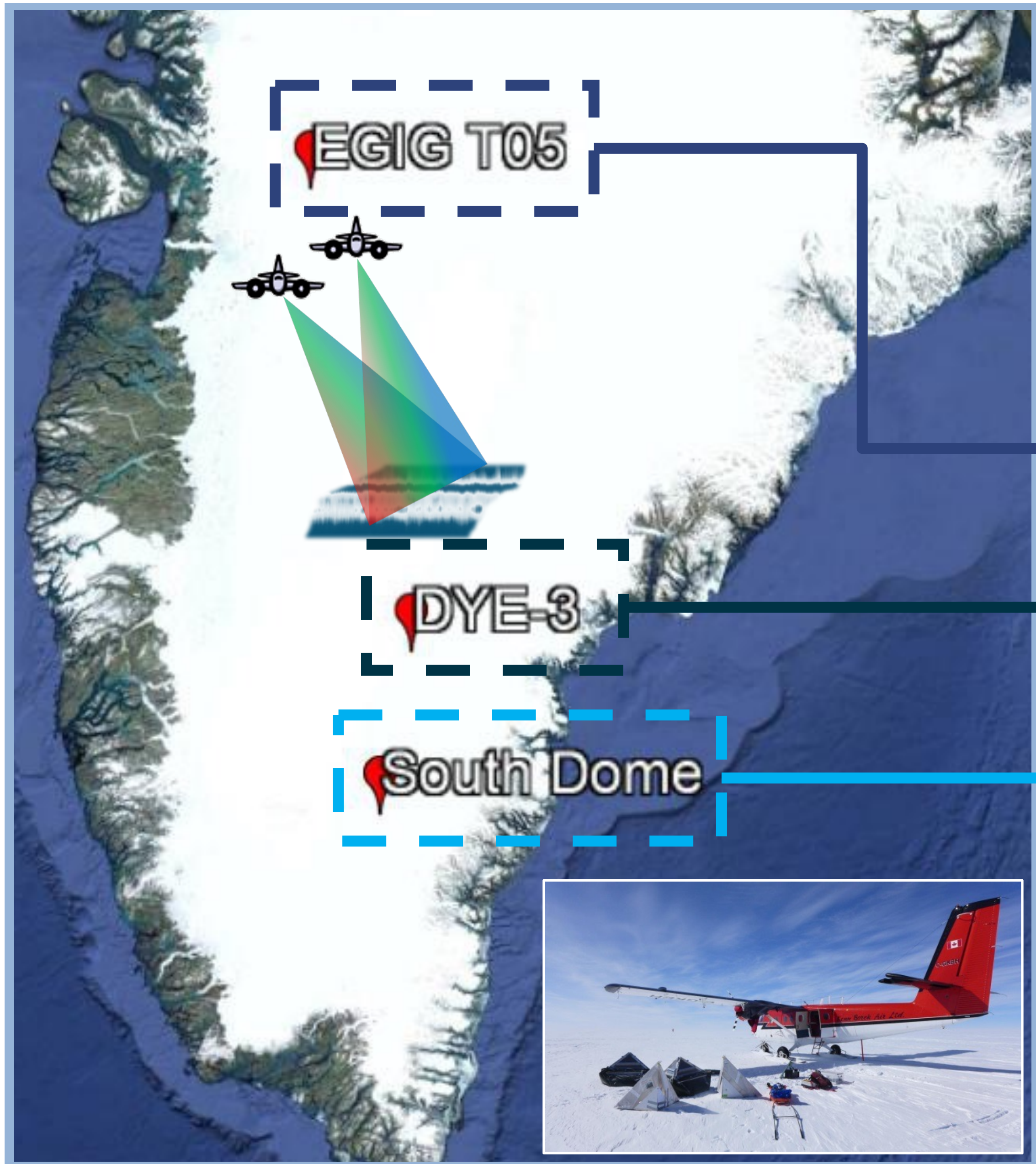
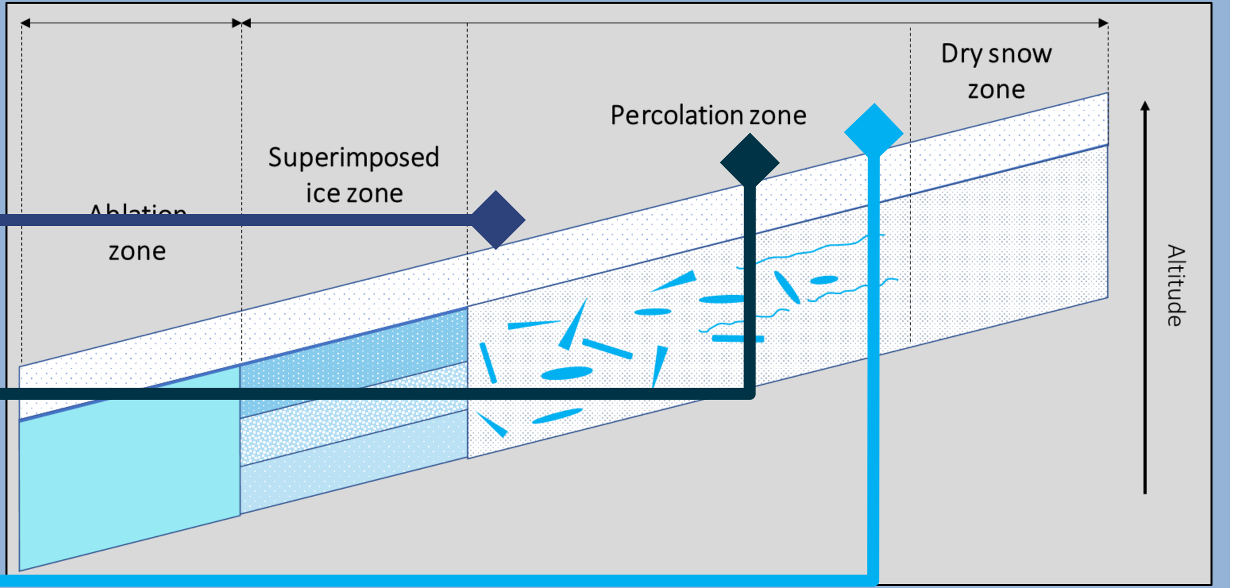
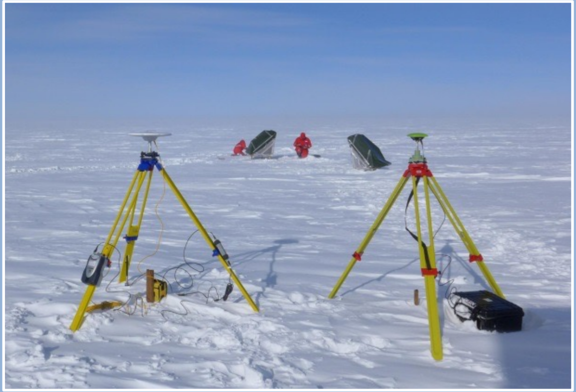
ARCTIC 2015 Campaign

F-SAR airborne sensor (DLR); P-, L-, S-, C-, X-band

Fully polarimetric & tomographic data





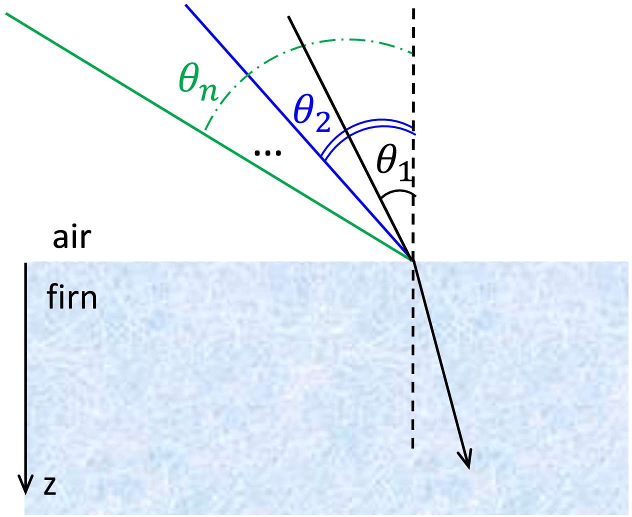


Pictures by Silvan Leinss

PolSAR Model: Multi-angular Configuration

Multiple incidence angles

$$\begin{cases} CPD_1 = CPD_{\theta_1} = f\left(\rho, DA, \sigma(z,\theta_1)\right) \\ CPD_2 = CPD_{\theta_2} = f\left(\rho, DA, \sigma(z,\theta_2)\right) \\ \dots \\ CPD_n = CPD_{\theta_n} = f\left(\rho, DA, \sigma(z,\theta_n)\right) \end{cases}$$

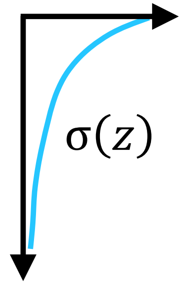


Model for $\sigma(z)$

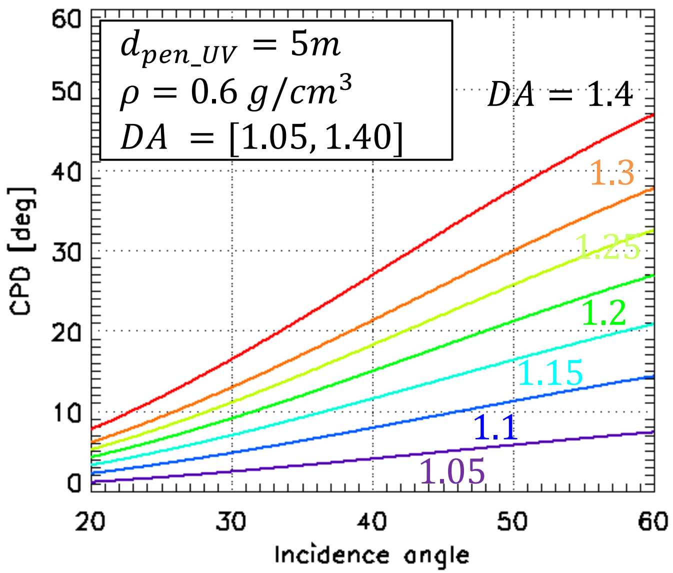
Uniform volume (UV) of scatterers with z

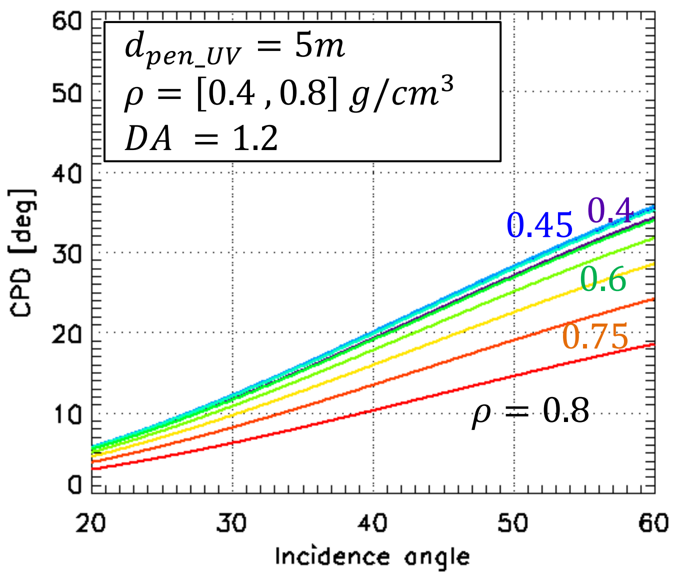
$$\sigma_{UV}(z) = e^{\frac{2k_e}{\cos\theta_r}z}$$

$$k_e = \frac{\cos\theta_r}{d_{pen}} = \text{const.}$$



3 unknowns → 3 incidence angles

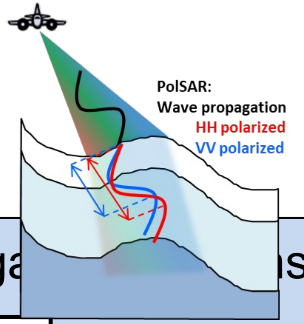




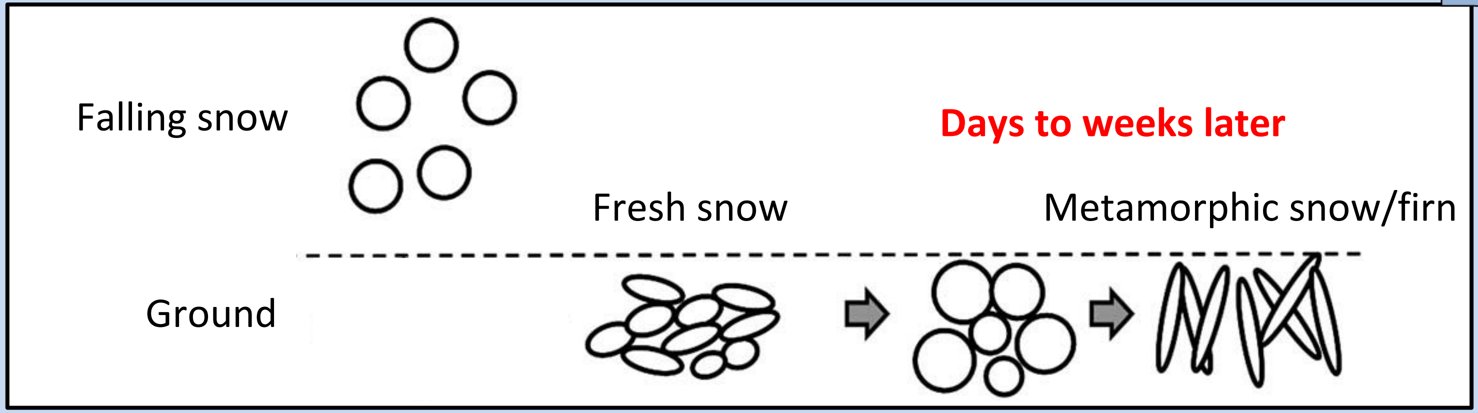
What if the UV assumption is not valid?

Using TomoSAR for $\sigma(z)$
→ 2 unknowns
→ Structure model independent

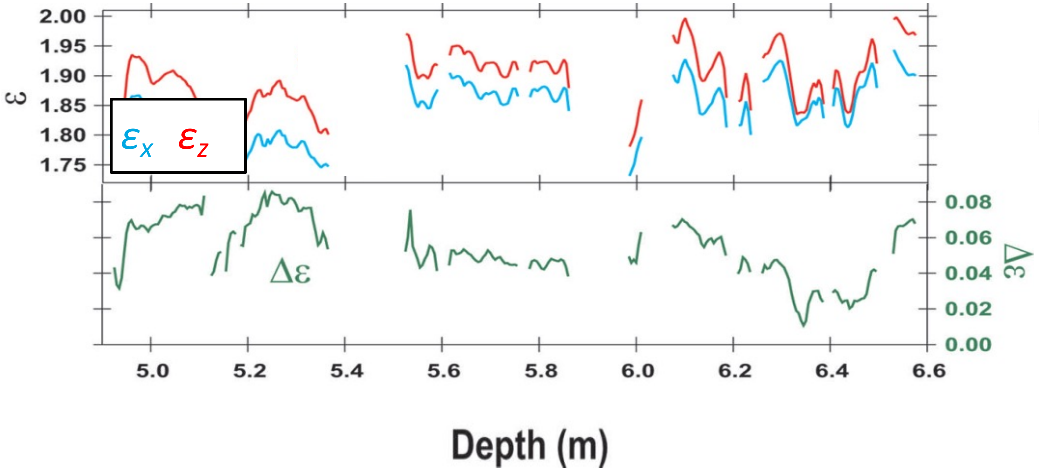
PolSAR Model: From CPD to Firn Properties

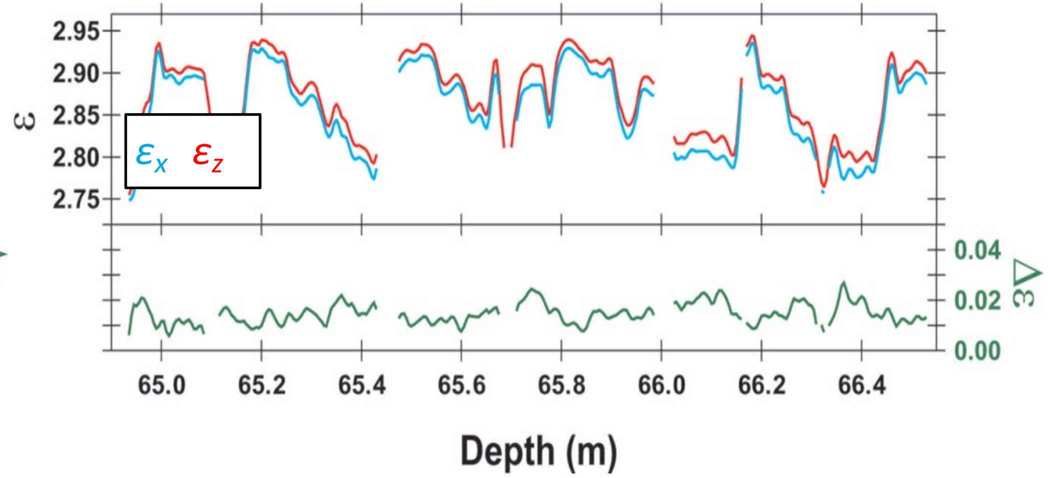


Firn is subject to (vertical temperature) **metamorphism** which generates vertical elonga[illegible]s

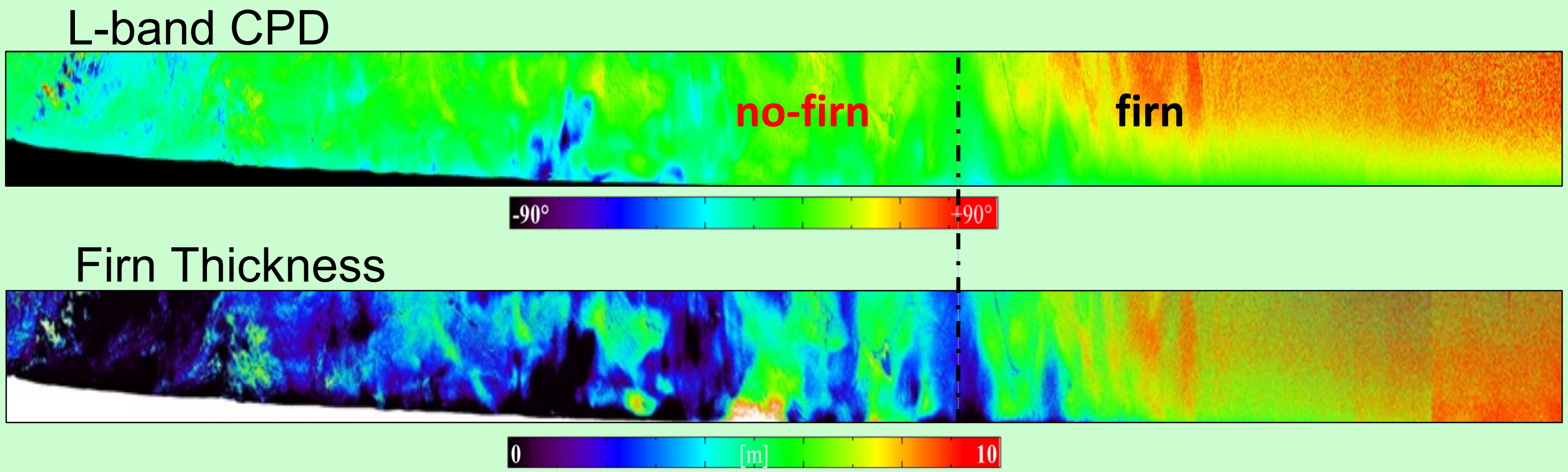


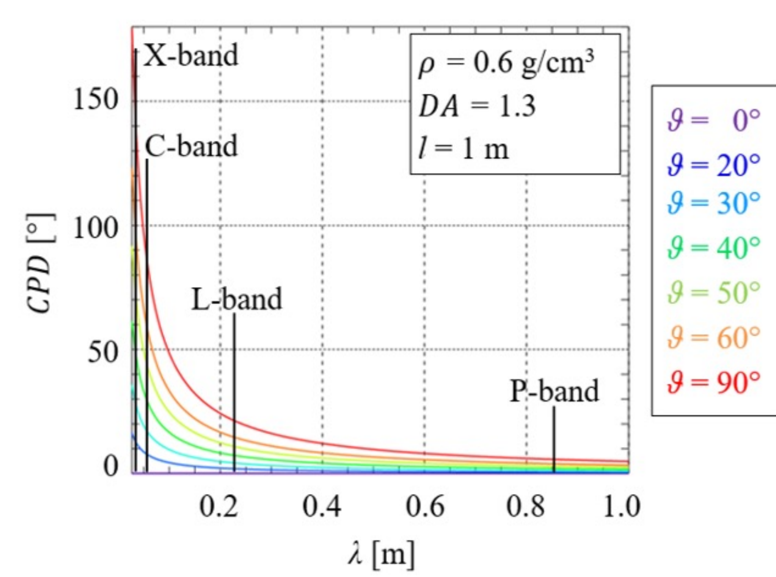
Structural anisotropy causes **dielectric anisotropy** (e.g. Fujita et al. 2014)





PolSAR Model: Multifrequency Sensitivity

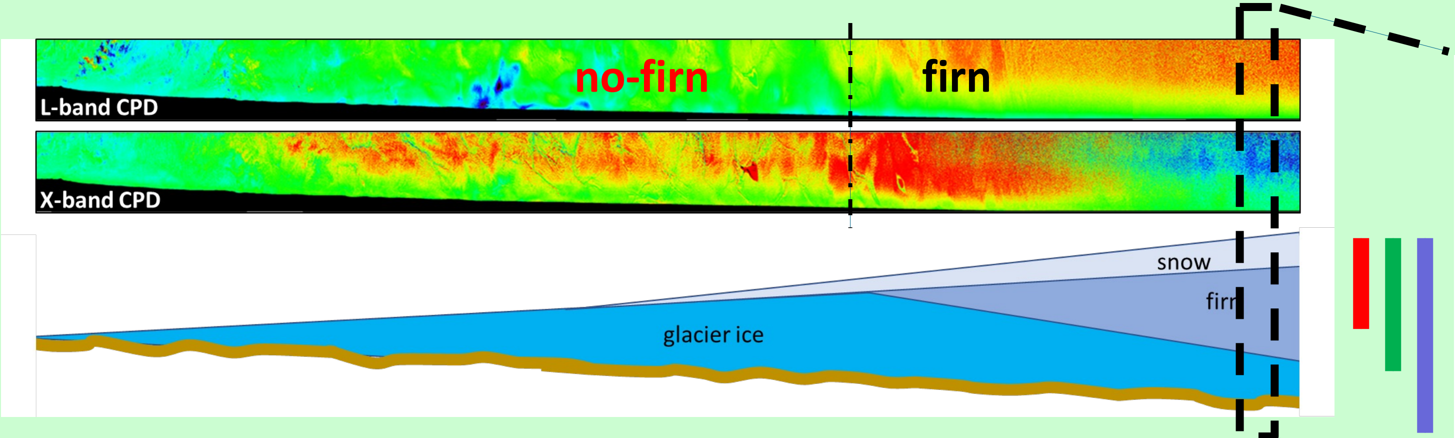


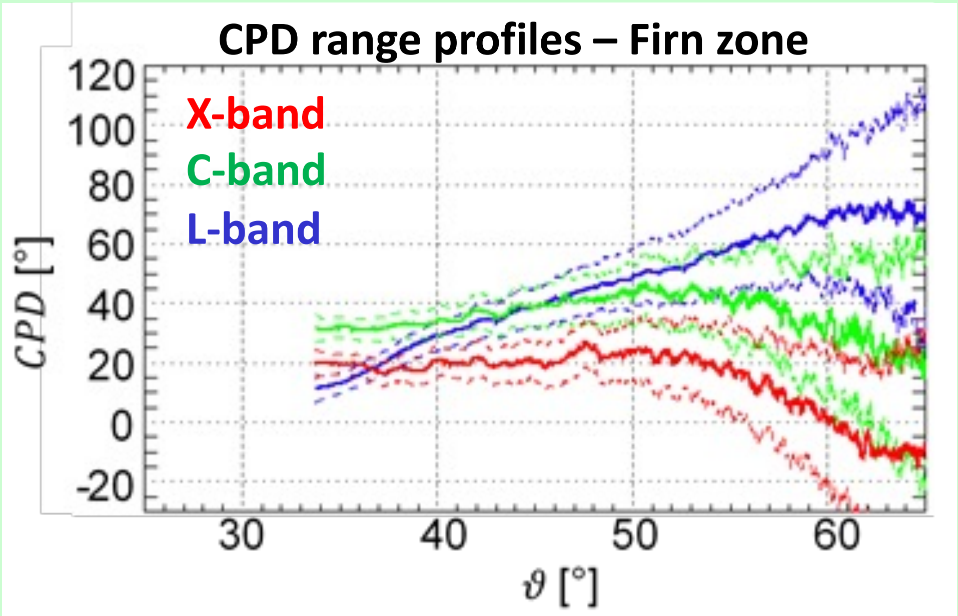


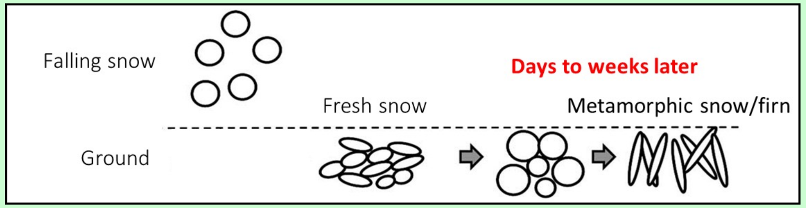
- Firn is assumed to be homogeneous (constant density, anisotropy and scattering) over depth
- CPD sensitivity vs. phase ambiguity

→ *multi-freq. configuration* for multi-layer scenarios (e.g. snow-over-firn)

PolSAR Model: Multifrequency Sensitivity







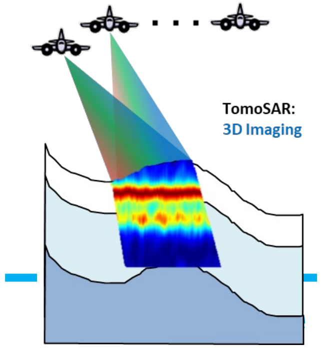
Fresh snow

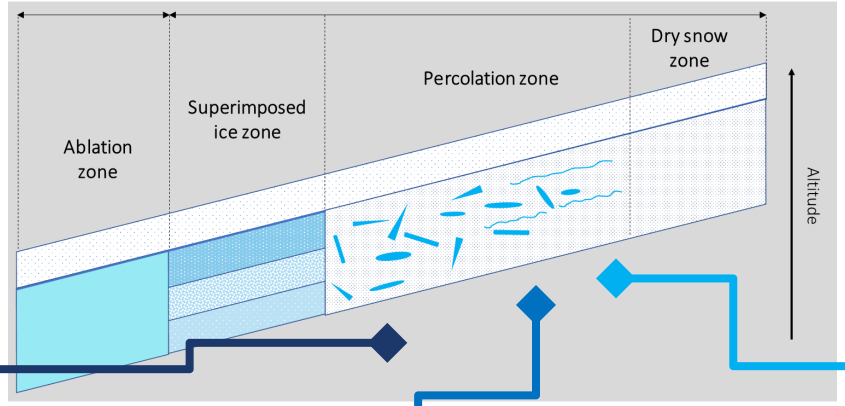
Horizontal grains → CPD < 0

Old Snow and Firn

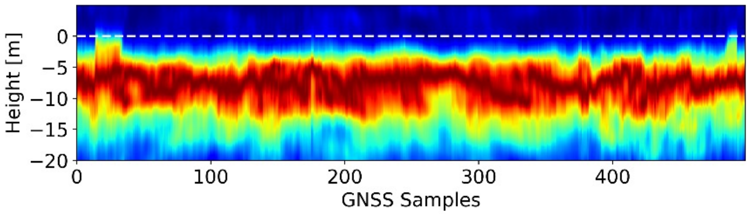
Vertical grains → CPD > 0

Tomography (P-band Examples)

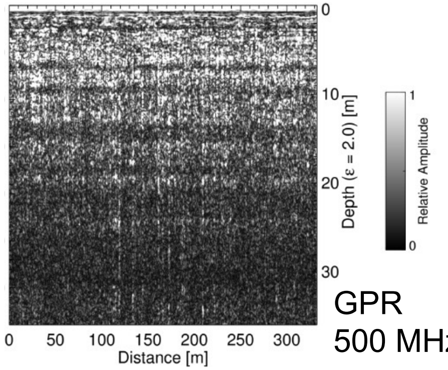




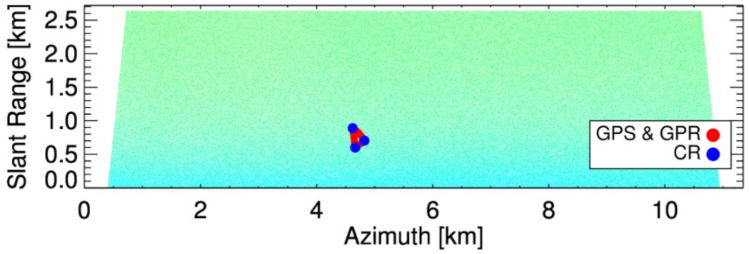
EGIG T05



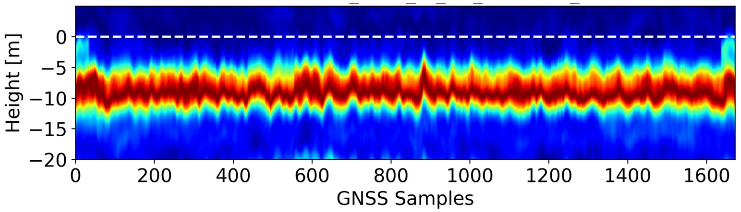
More melting
Uniform scattering
Higher density

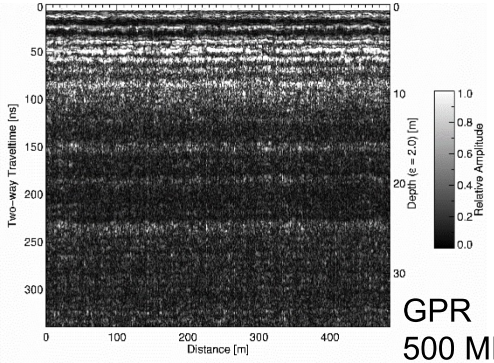


GPR
500 MHz

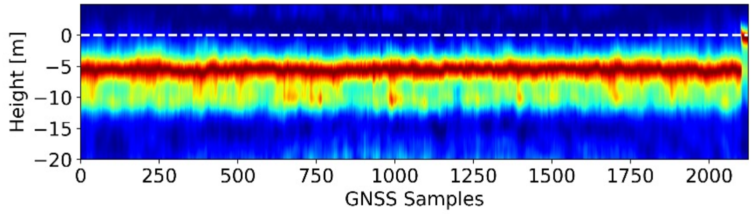


DYE-3

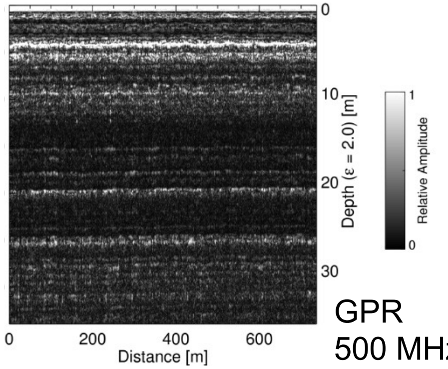




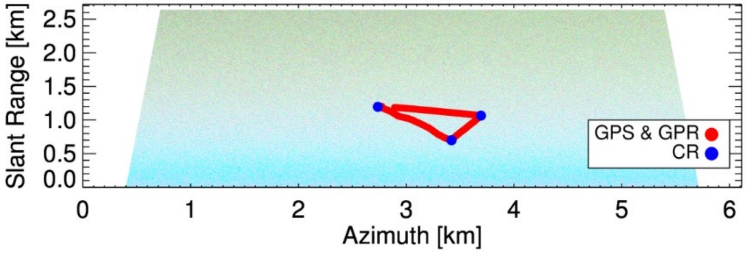
GPR
500 MHz



Less melting
Distinct layers
Lower density



GPR
500 MHz

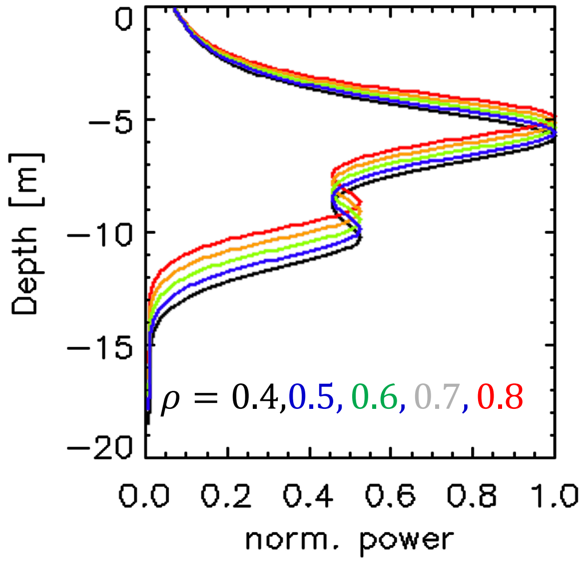


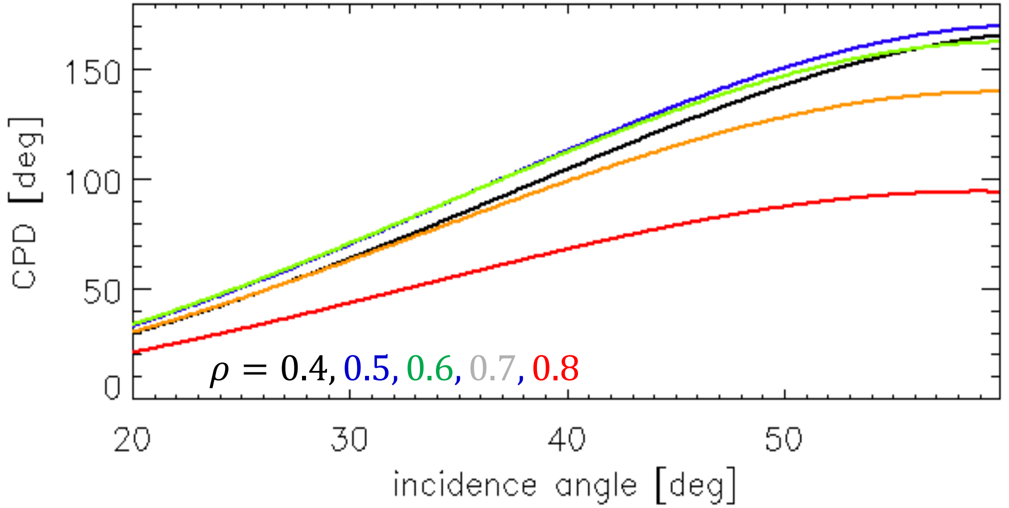
PolSAR CPD + Tomo Scattering Structure

- TomoSAR for a more realistic vertical scattering structure: $\sigma_{UV}(z) \Rightarrow \sigma_{tomo}(z)$

$$CPD_{tomo} = \boldsymbol{f}(\boldsymbol{DA}, \boldsymbol{\rho})$$

- Density ρ scales $\sigma_{tomo}(z)$ and CPD:

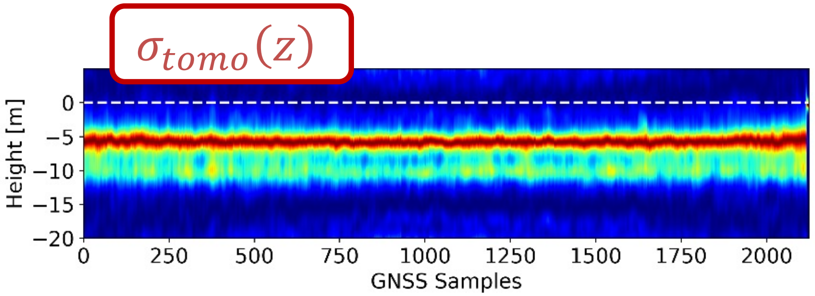


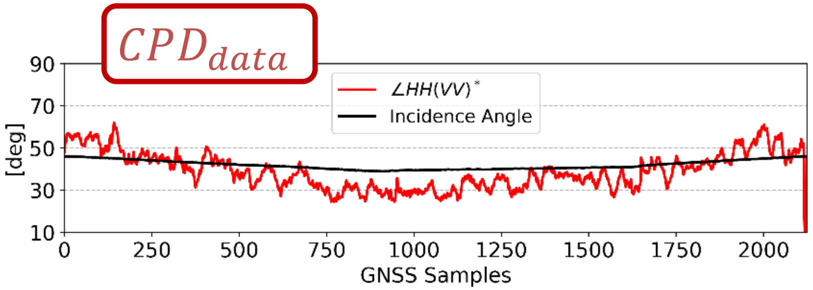


→ Angular diversity to solve for 2 unknowns

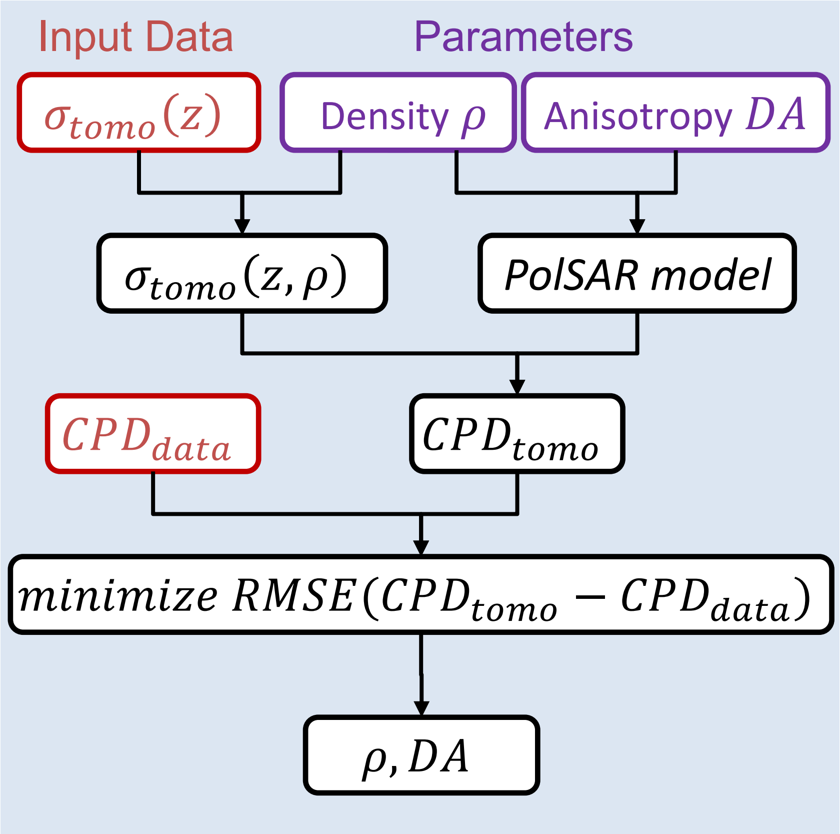
CPD and Tomo: Numerical Density Inversion

Input Data
$\sigma_{tomo}(z)$ at $HH(VV)^*$ channel
$CPD_{data} = \angle HH(VV)^*$

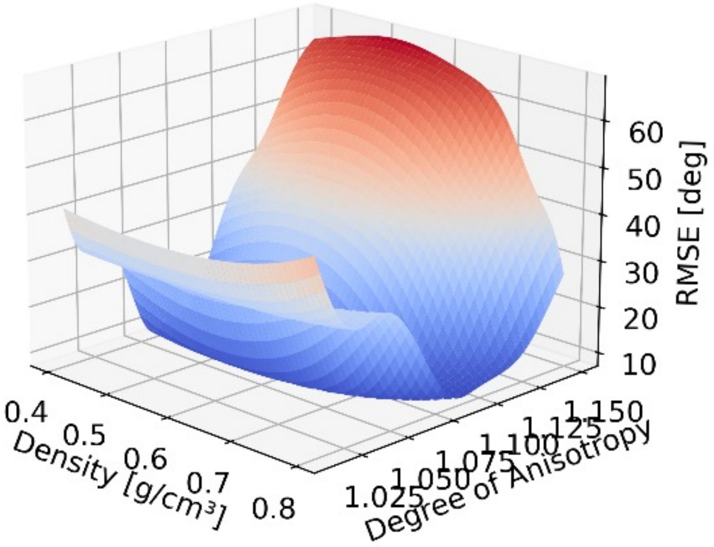




Flow Chart
Minimization of data vs model
over all samples (incidence angles)

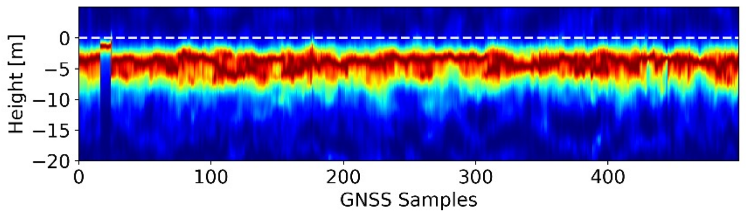


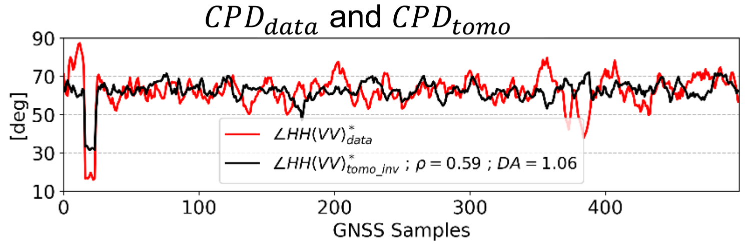
Cost Function
$RMSE(CPD_{data} - CPD_{tomo})$
→ Best solution of ρ and DA



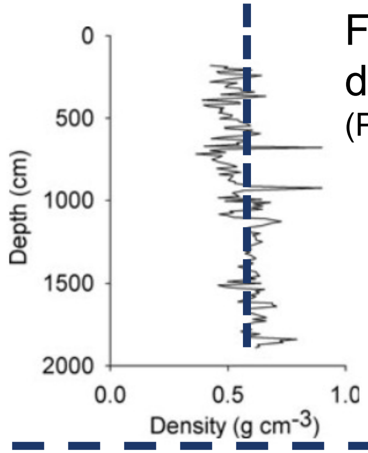
Combination (L-band Results)

EGIG T05

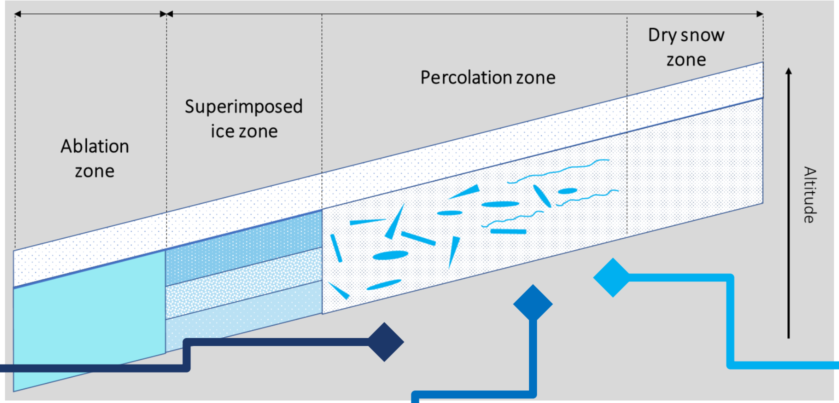




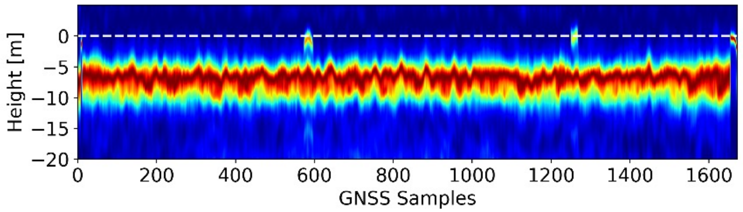
$$\rho = 0.59\ g/cm^3$$
$$DA = 1.06$$

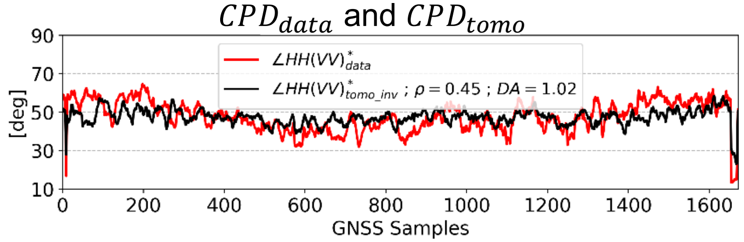


Firn core density (Parry et al., 2007)



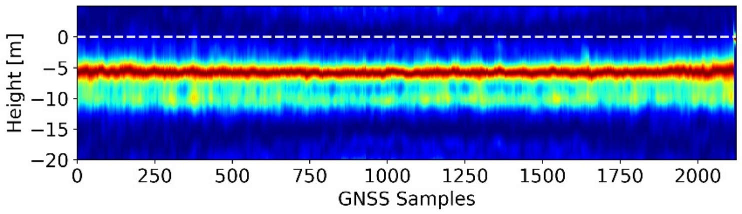
DYE-3

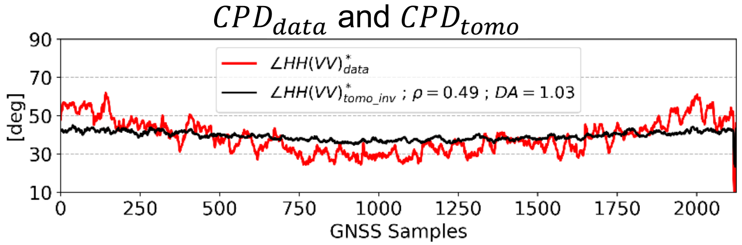




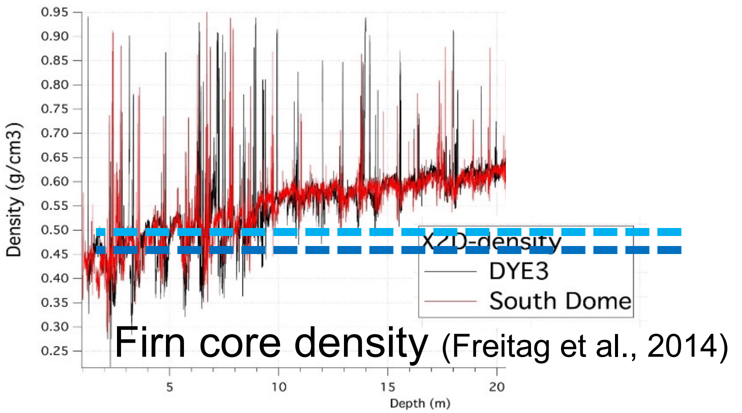
$$\rho = 0.45\ g/cm^3$$
$$DA = 1.02$$

South Dome





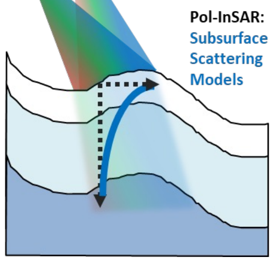
$$\rho = 0.49\ g/cm^3$$
$$DA = 1.03$$



Firn core density (Freitag et al., 2014)

V. Parry *et al.*, "Investigations of meltwater refreezing and density variations in the snowpack and firn within the percolation zone of the Greenland ice sheet," *Annals of Glaciology*, vol. 46, pp. 61–68, 2007.

J. Freitag, S. Kipfstuhl, S. Hoerz, L. Eling, B. Vinther, and T. Popp, "Melt layer statistic of two firn cores recently drilled at Dye3 and South Dome in the dry snow zone of Southern Greenland," *presented at the EGU General Assembly*, Vienna, Austria, Apr./May 2014.



Random Uniform Volume + 1 Layer

- 3 structure parameters: z_{ul}, d_{pen}, z_{lay}
- 3 polarimetric parameters: $m(\vec{w})$

$$\gamma = \frac{\gamma_{Vol}(z_{ul}, d_{pen}) + m(\vec{w})e^{ik_{zVol}z_{lay}}}{1 + m(\vec{w})}$$

Triple Baseline Full-Pol Inversion
(Alonso-Gonzalez et al., 2018)

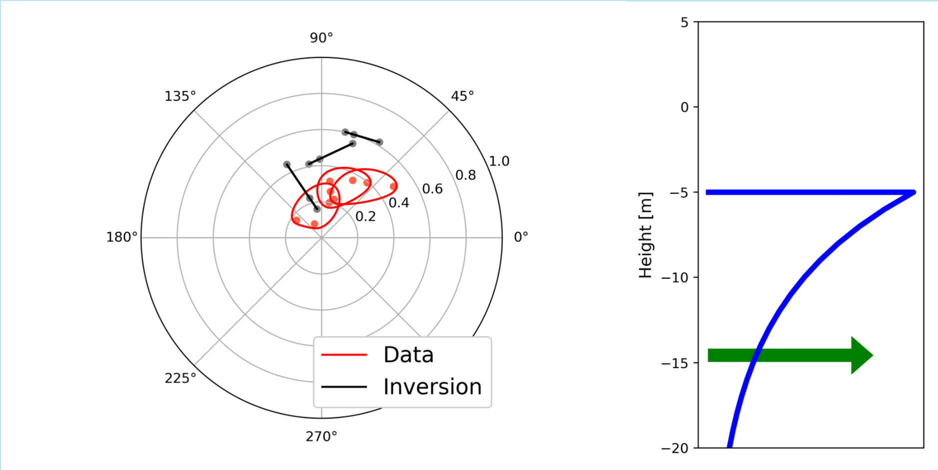
Coherences → from model

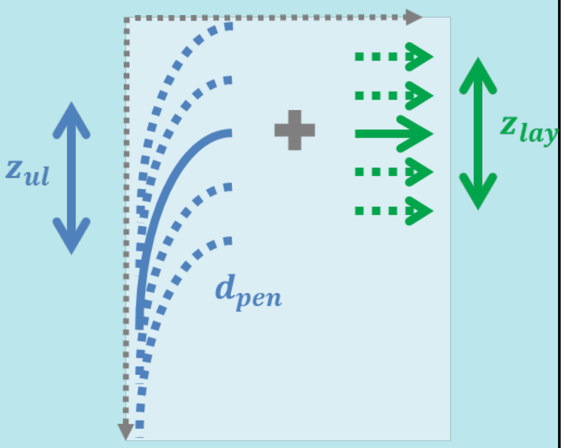
⇩

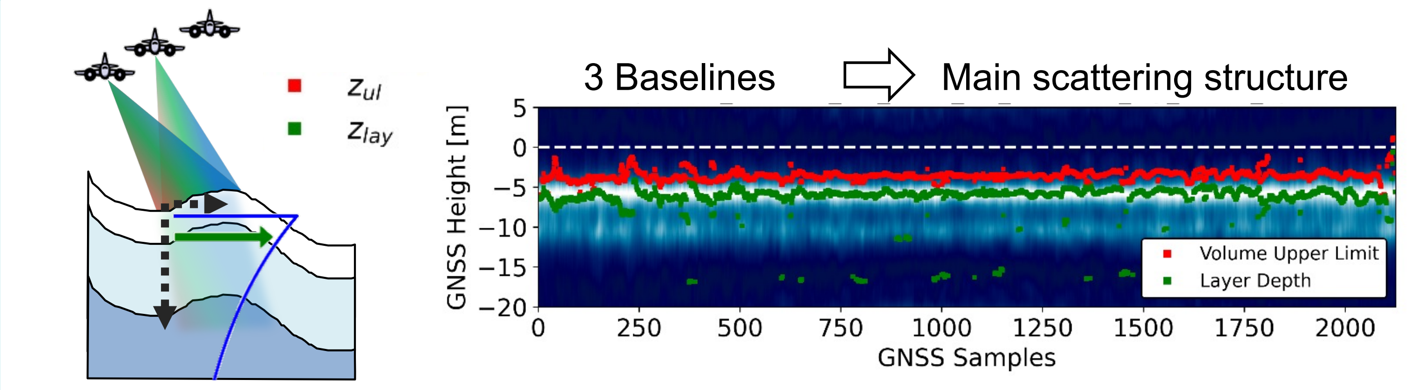
Polarimetry → estimated

⇩

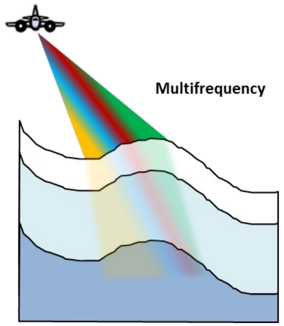
Pol-InSAR matrix → fit to data



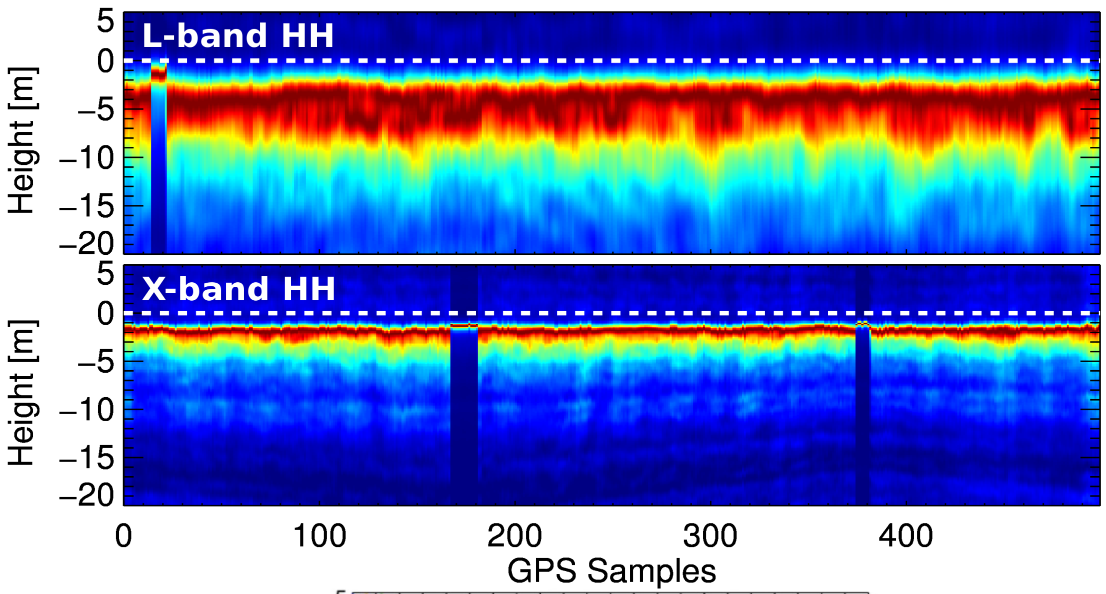


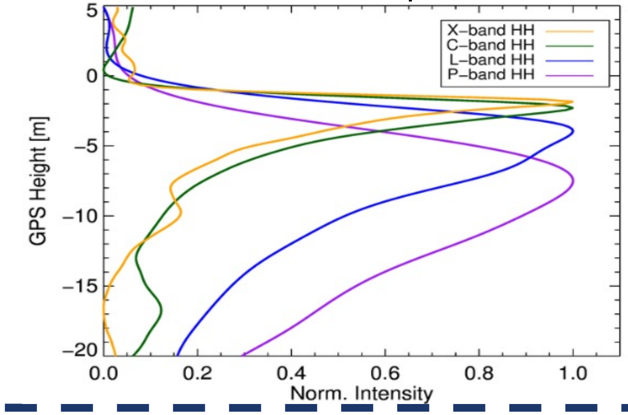


Tomography: Multifrequency Information Content

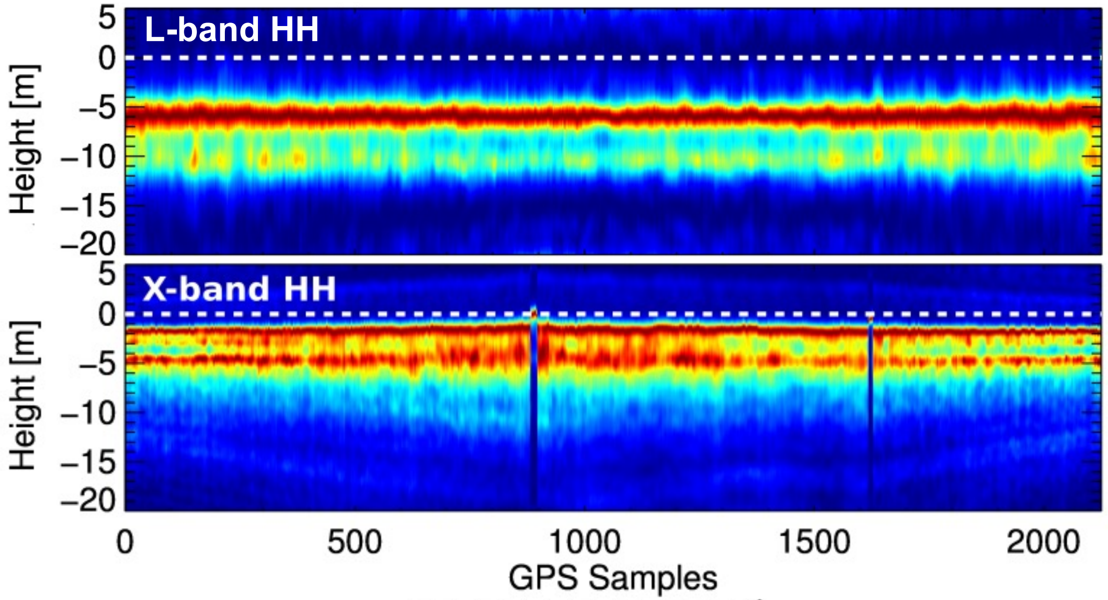


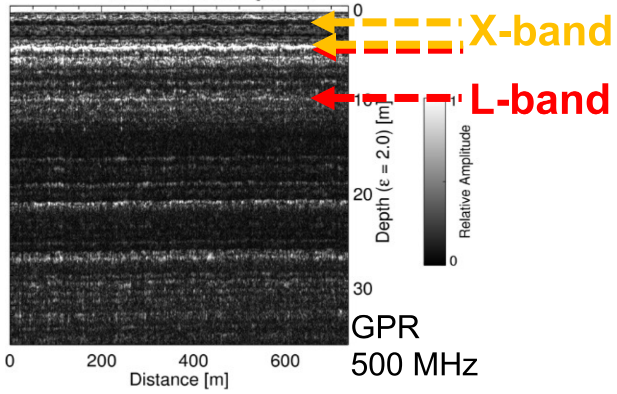
EGIG T05 / Lower Percolation





South Dome / Upper Percolation



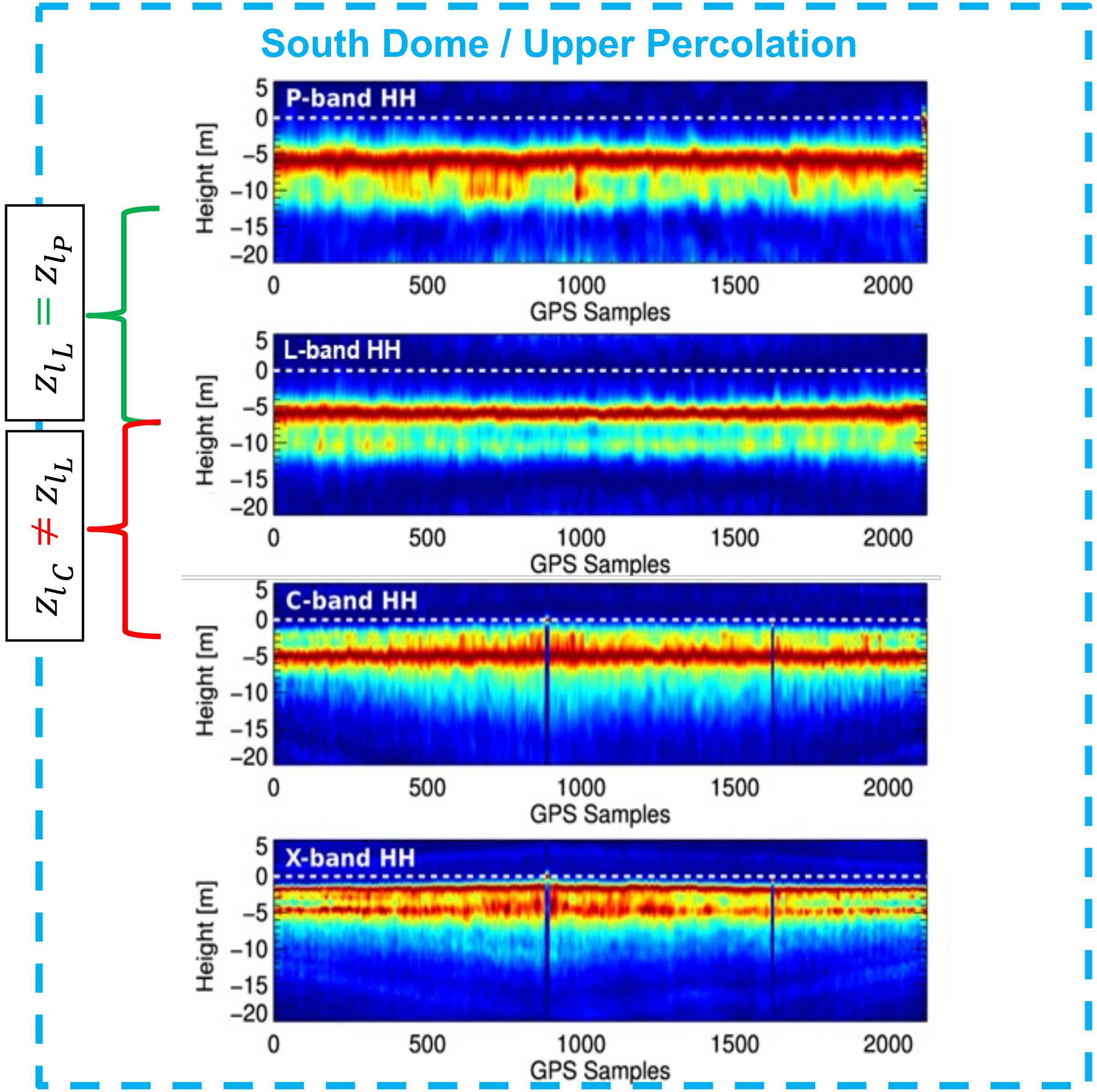


Tomography: Multifrequency Information Content

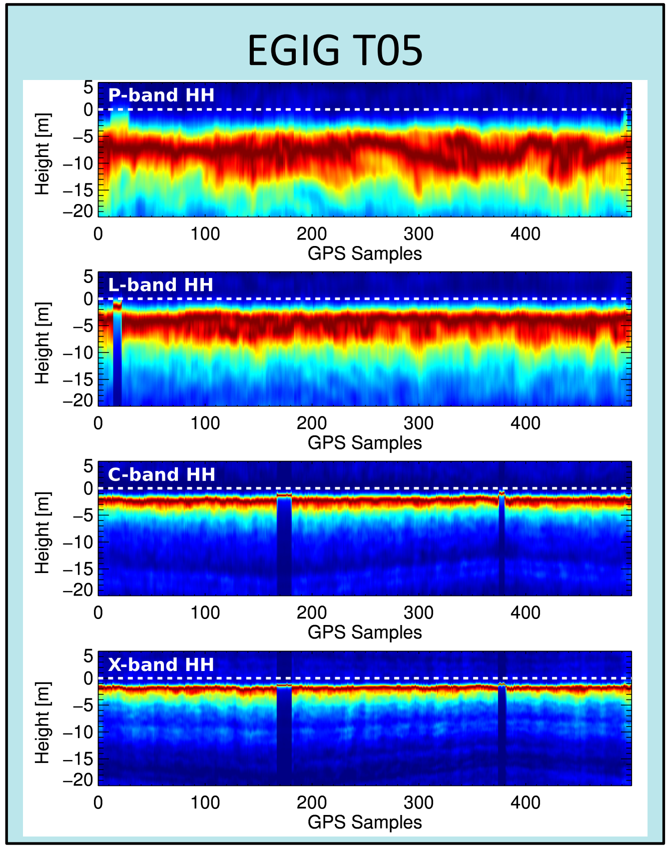
- Frequencies:
  - Different penetration depths
  - Different scatterers
- Penetration:
  - P < L < C < X
  - HV < HH,VV

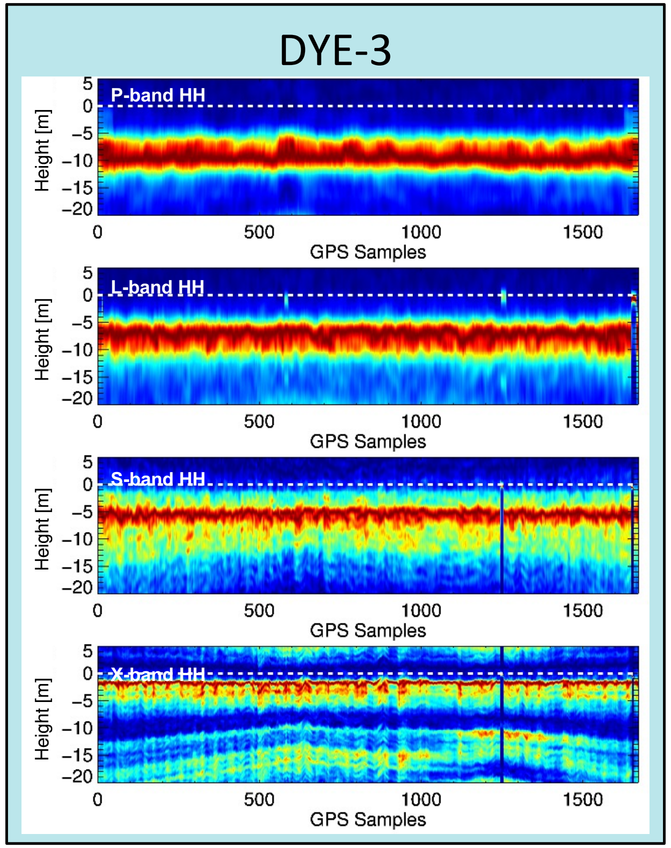
→ More complete picture of subsurface structure

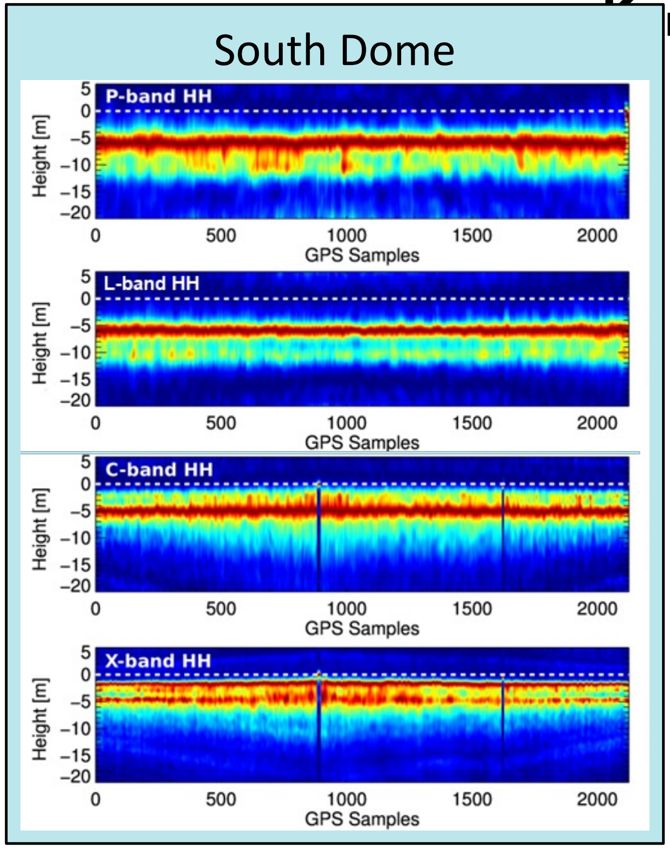
- Limited observation spaces: Pol-InSAR model
  - Same volume "shape"
  - Dirac deltas partly transferable



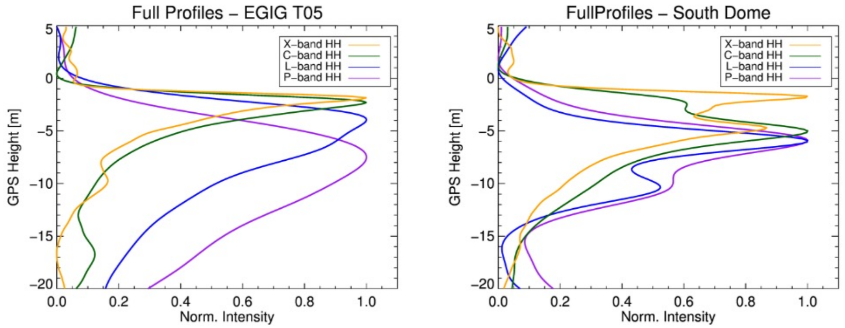
Multi-frequency Scattering Structure: Tomography





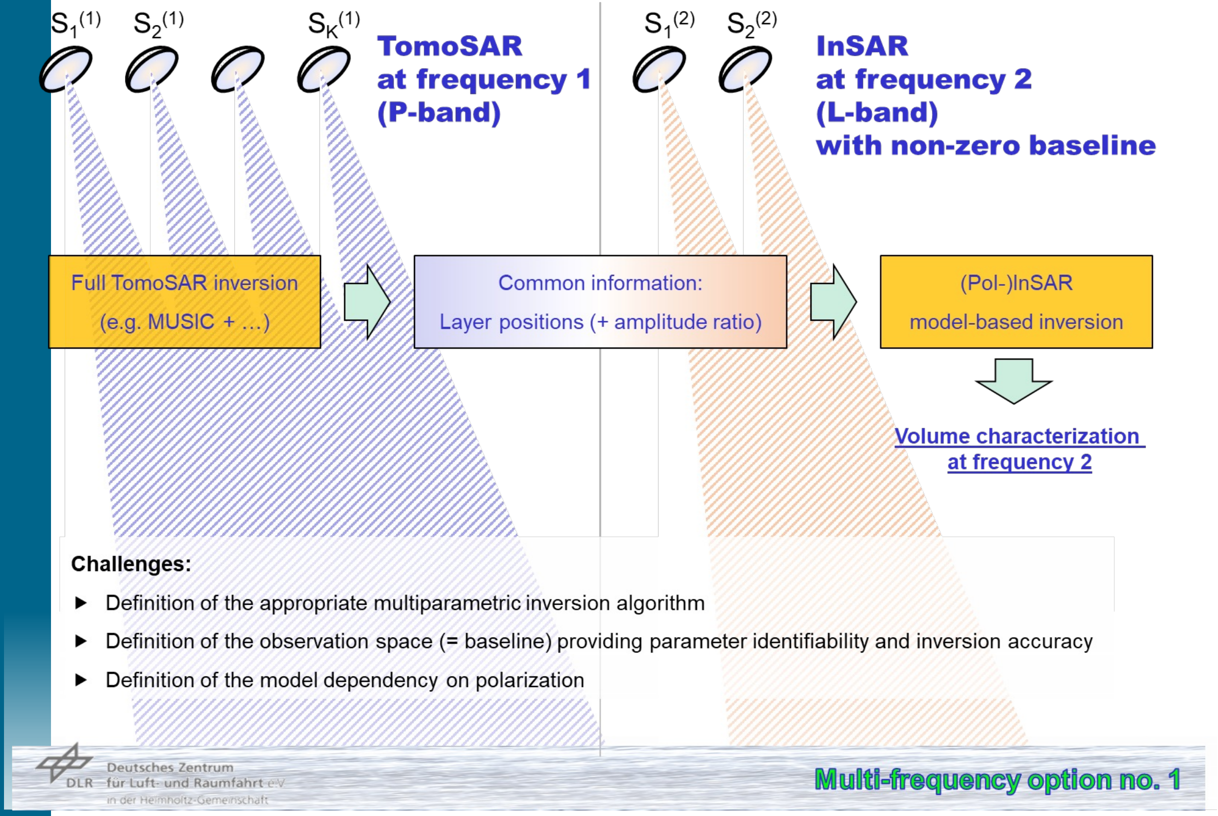


- Different frequencies:
  → Common and different layers and depths
  → “Volume” part: similar structure functions (absolute or only shape?)
- **Which parts of the scattering structure are common / different?**
  → multifrequency model

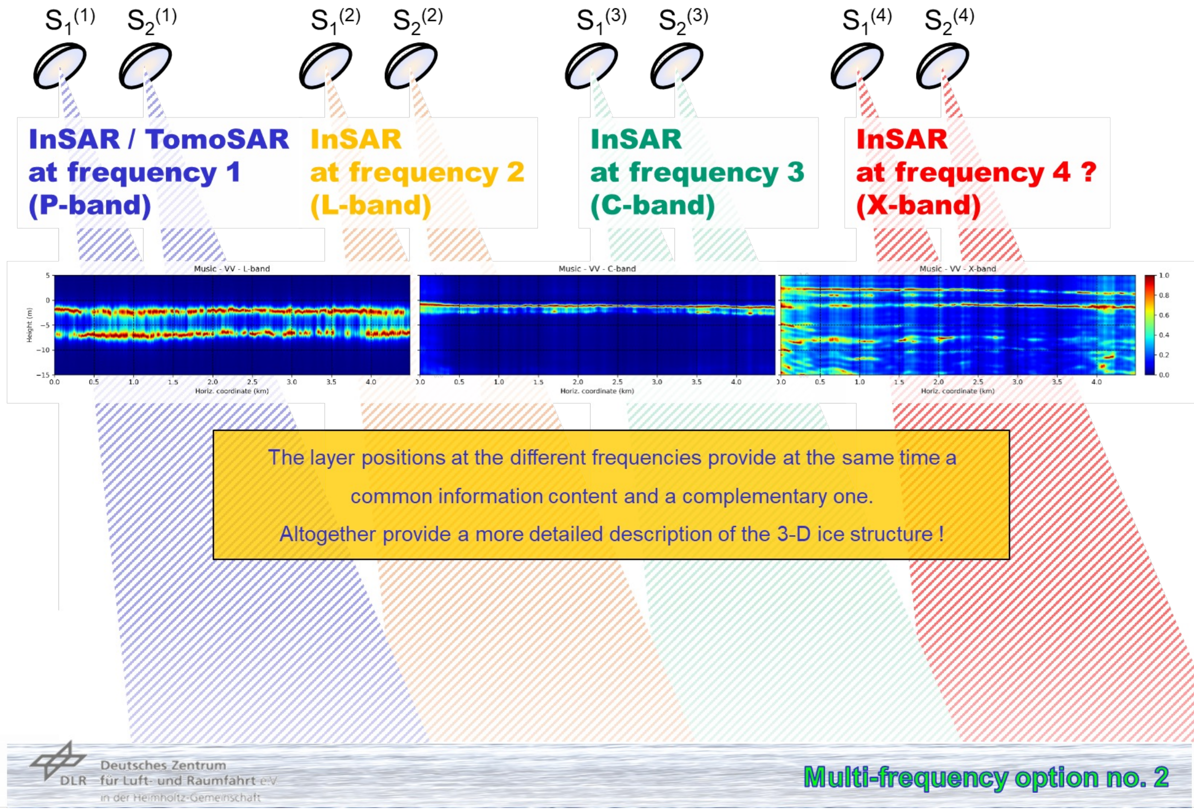


Multispectral Pol-InSAR inversion: 2 ideas

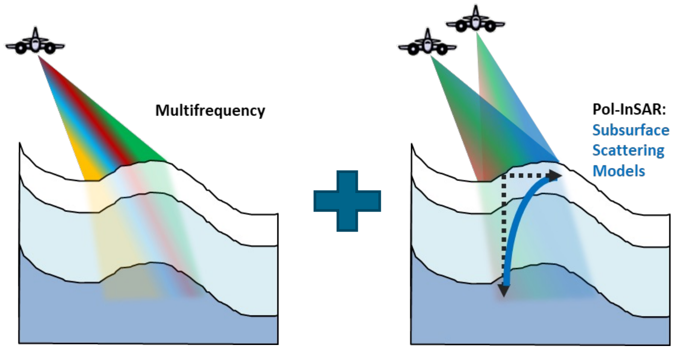
- Layer positions z_j from P-band MUSIC to support the (Pol-)InSAR model inversion at L-band
- Output: Volume parameters and layer-to-volume ratios → (glaciological) application and interpretation not clear



- Combination of Layer positions z_j from different frequencies with limited baselines
- Output: More detailed and complete 3-D structure information!



Idea: Transfer Layer Positions across Frequencies



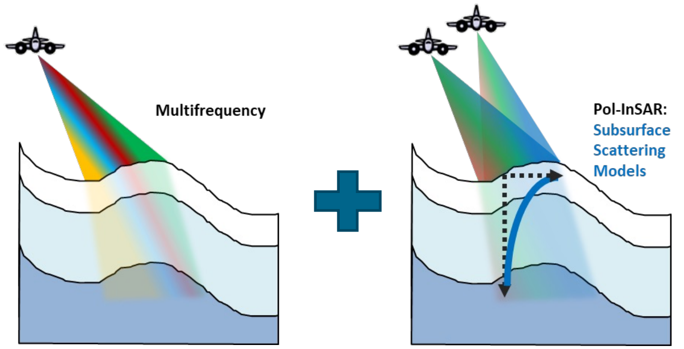
$$\gamma = e^{ik_z z_0} \frac{\gamma_{Vol}(d_{pen}, z_{ul}, \vec{w}) + \sum_{j=1}^{N} m_j(\vec{w}) e^{ik_{zVol} z_j}}{1 + \sum_{j=1}^{N} m_j(\vec{w})}$$

$m_j(\vec{w})$: layer-to-volume ratio
z_j: layer position
N: number of layers
d_{pen}: one-way penetration depth
z_{ul}: upper limit of volume

- Expressing the model as a sum of uncorrelated scattering elements (2 layers and 1 volume)
- MB covariance matrix: $\boldsymbol{T} = \boldsymbol{vR_v} + \boldsymbol{m_1 R_{l1}} + \boldsymbol{m_2 R_{l2}}$ with structure matrices R.
- Linear equation system ($r = U\alpha - y$): $\min||vR_v + m_1 R_{l1} + m_2 R_{l2} - \mathrm{T}||_2^2$

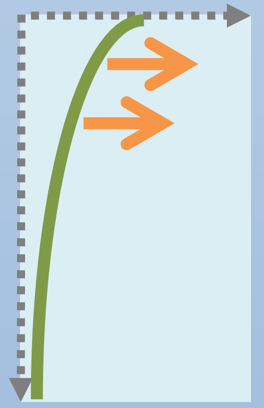
$$U = [vec(R_v), vec(R_{l1}), vec(R_{l2})]$$
$$y = vec(\mathrm{T})$$
$$\alpha = [v, m_1, m_2] = (U^H U)^{-1} U^H y$$

Idea: Transfer Layer Positions across Frequencies



$$\gamma = e^{ik_z z_0} \frac{\gamma_{Vol}(d_{pen}, z_{ul}, \vec{w}) + \sum_{j=1}^{N} m_j(\vec{w}) e^{ik_{zVol} z_j}}{1 + \sum_{j=1}^{N} m_j(\vec{w})}$$

$m_j(\vec{w})$: layer-to-volume ratio
z_j: layer position
N: number of layers
d_{pen}: one-way penetration depth
z_{ul}: upper limit of volume



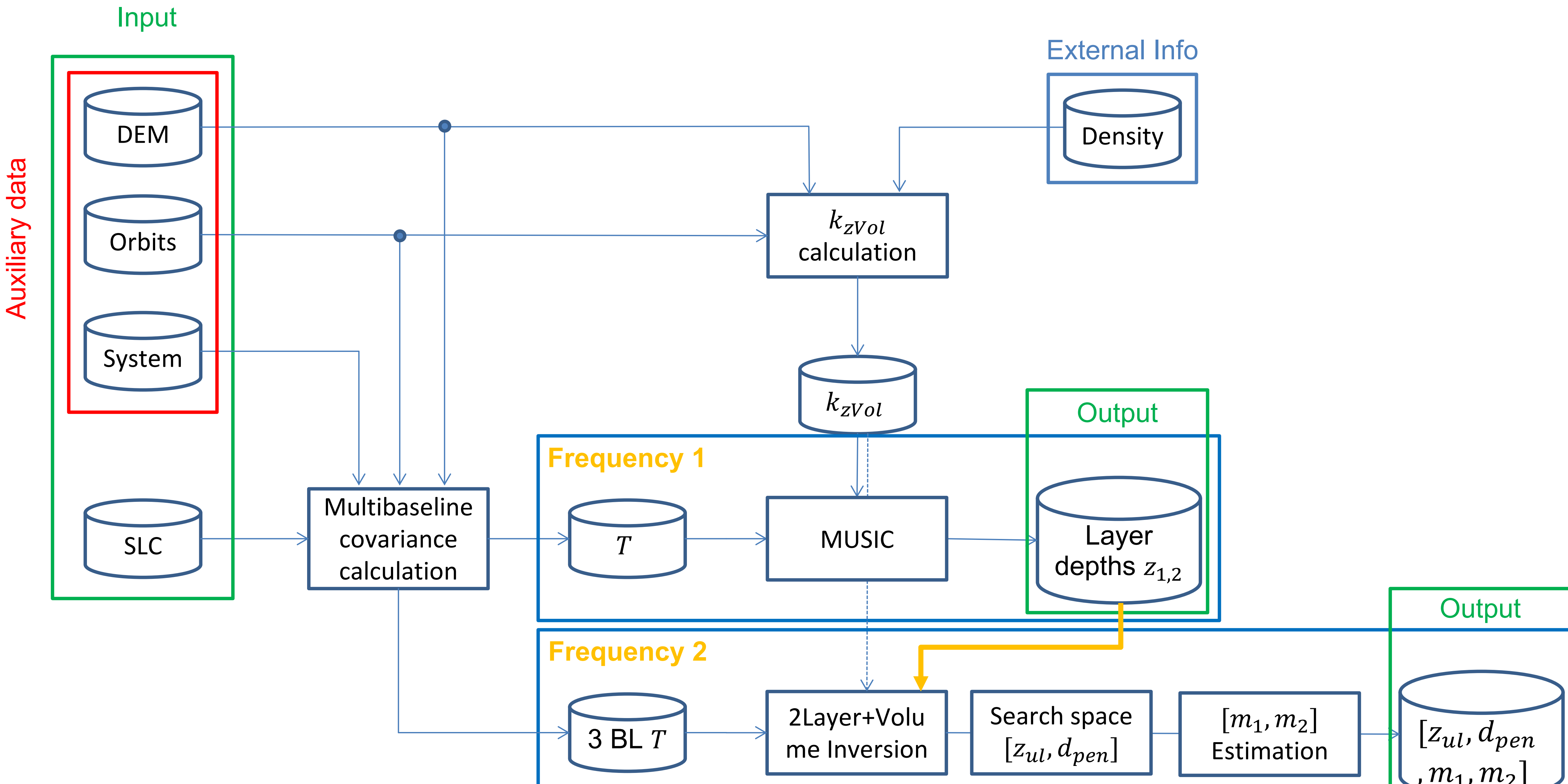
- Model: Sum of uncorrelated scattering elements (2 layers and 1 volume)
- MB covariance matrix: $\boldsymbol{T} = \boldsymbol{vR_v} + \boldsymbol{m_1R_{l1}} + \boldsymbol{m_2R_{l2}}$ with structure matrices R.
- Linear equation system: $\min||vR_v + m_1R_{l1} + m_2R_{l2} - \mathrm{T}||_2^2$

→ **Search space:** $[z_{ul}, d_{pen}]$

→ $[m_1, m_2]$ **estimated from data**

→ **3 Baseline** model inversion at L-band of higher complexity than possible with single-frequency

MS InSAR inversion flow chart



P-band MUSIC layers for L-band 2-layer+UV inversion

<u>3 Baseline</u> model Inversion based on Frobenius norm between the **MB covariance matrices.**

Search space: $[z_{ul}, d_{pen}]$.

$[m_1, m_2]$ **from data**:

HH VV HV

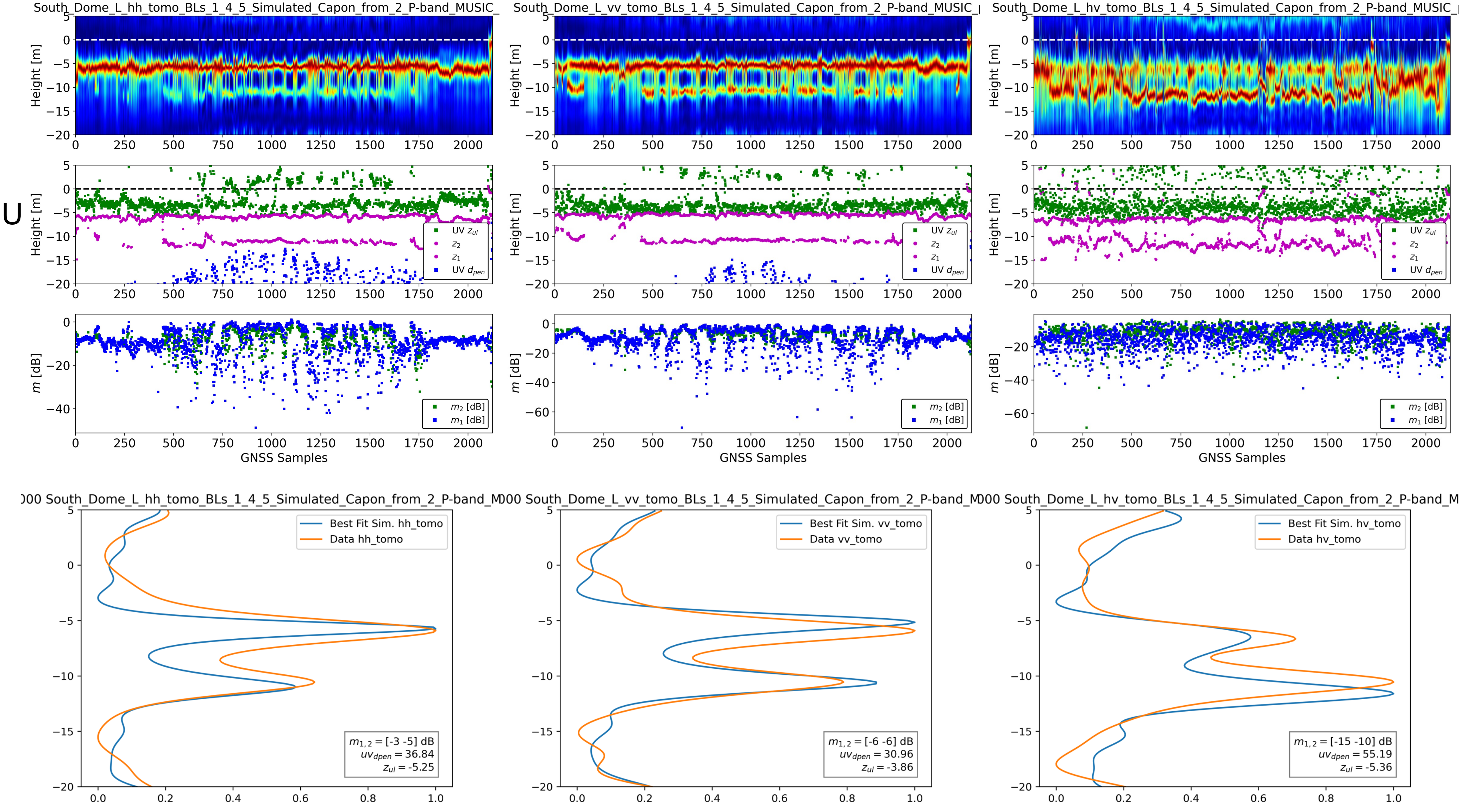
Simulated Capon:

$z_{1,2}$ from P-band MU

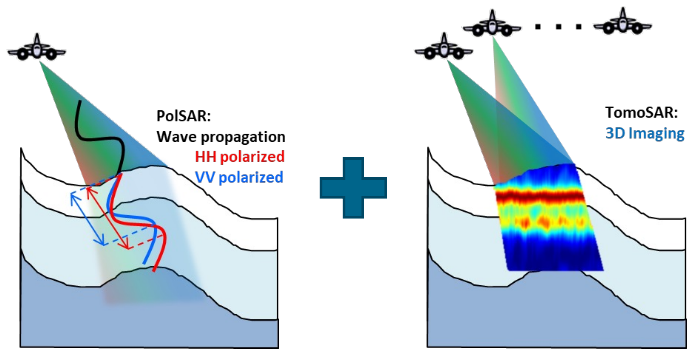
Inverted z_{ul}, d_{pen}

Inverted m_1, m_2:

Example profile (Model inversion vs. data)



Conclusions (PolSAR CPD Model + Tomo Structure)



Pros

- Direct link to density through physical model.
- No model calibration, empirical parameters or a priori information required.
- Feasible at multilooked SAR resolution (e.g. ~100m for spaceborne SAR).

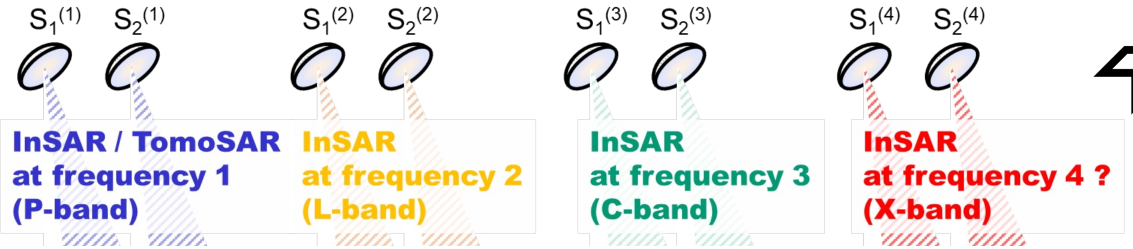
Cons

- Bulk density, strong approximation of real firn structure.
- Purely based on vertical anisotropy from temperature gradient metamorphism.
- SAR tomography not available from space yet (→ *BIOMASS* ?)

Open Questions:

- How to achieve a benefit for glaciological researchers?
- Ideas for more realistic firn parameterization?
- Potential to integrate firn densification models?
- Projection to spaceborne SAR?
- Sensitivity analysis of model and acquisition parameters not yet finished.
- Potential to use Pol-InSAR to estimate $\boldsymbol{\sigma}(\boldsymbol{z})$ with less baselines.

Conclusions (Multifrequency)



Pros

- Land ice structure retrieval with Multispectral PolInSAR shows great potential
- Added information content of frequencies is very clear (different penetration, different layers)
- Detection of refrozen melt layer possible with dual-baseline (Pol-)InSAR
- P-band Tomo to initialize L-band Pol-InSAR model inversion (higher complexity than single-frequency possible)

Cons

- Still many open research questions also in single-frequency case
- Across-track baselines without temporal decorrelation required
- Challenging applicability to spaceborne missions

Open Questions:

Outlook

Next steps:

- Sensitivity analysis in terms of baselines (Pol-InSAR model inversion) and incidence angles
- Connection to glaciological/geophysical models of firn structure (firn densification models)

Open Questions:

- How to achieve a benefit for glaciological researchers?
- Ideas for more realistic firn parameterization?
- Potential to integrate firn densification models?
- Projection to spaceborne SAR?
- Sensitivity analysis of model and acquisition parameters not yet finished.
- Potential to use Pol-InSAR to estimate $\boldsymbol{\sigma}(\boldsymbol{z})$ with less baselines.