



# SPACEBORNE FDM MIMO SAR TOMOGRAPHY

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## The number of SAR sensors in space is clearly destined to increase in the next years

### Examples of commercial SAR constellations (as of September 2022)

- **IceEye:** *We've launched 20 satellites since 2018 with 10+ more planned for 2022 and beyond.* <https://www.iceeye.com/satellite-missions>
- **Synspective:** *We aims to build a constellation of 6 satellites by 2023 and 30 satellites by the late 2020s.* <https://synspective.com/satellite/>
- **Capella Space:** *... is developing the Capella X-SAR (Synthetic Aperture Radar) constellation consisting of 36 microsattellites... Two microsattellites were launched in December 2018 ... joined by six operational Capella microsattellites as of January 2022.* <https://www.eoportal.org/satellite-missions/capella-x-sar>

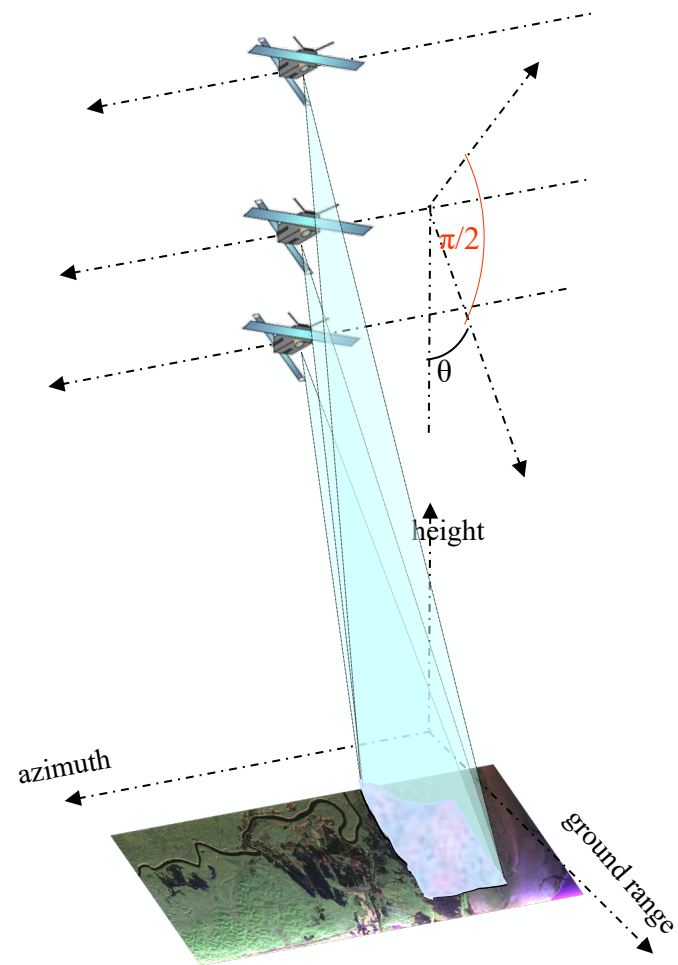
### ***Does that make a new SAR paradigm?***

The main goal is to improve the current capabilities of SARs systems, primarily in terms of revisit rate and coverage, to address applications that guarantee immediate returns and leverage consolidated concepts

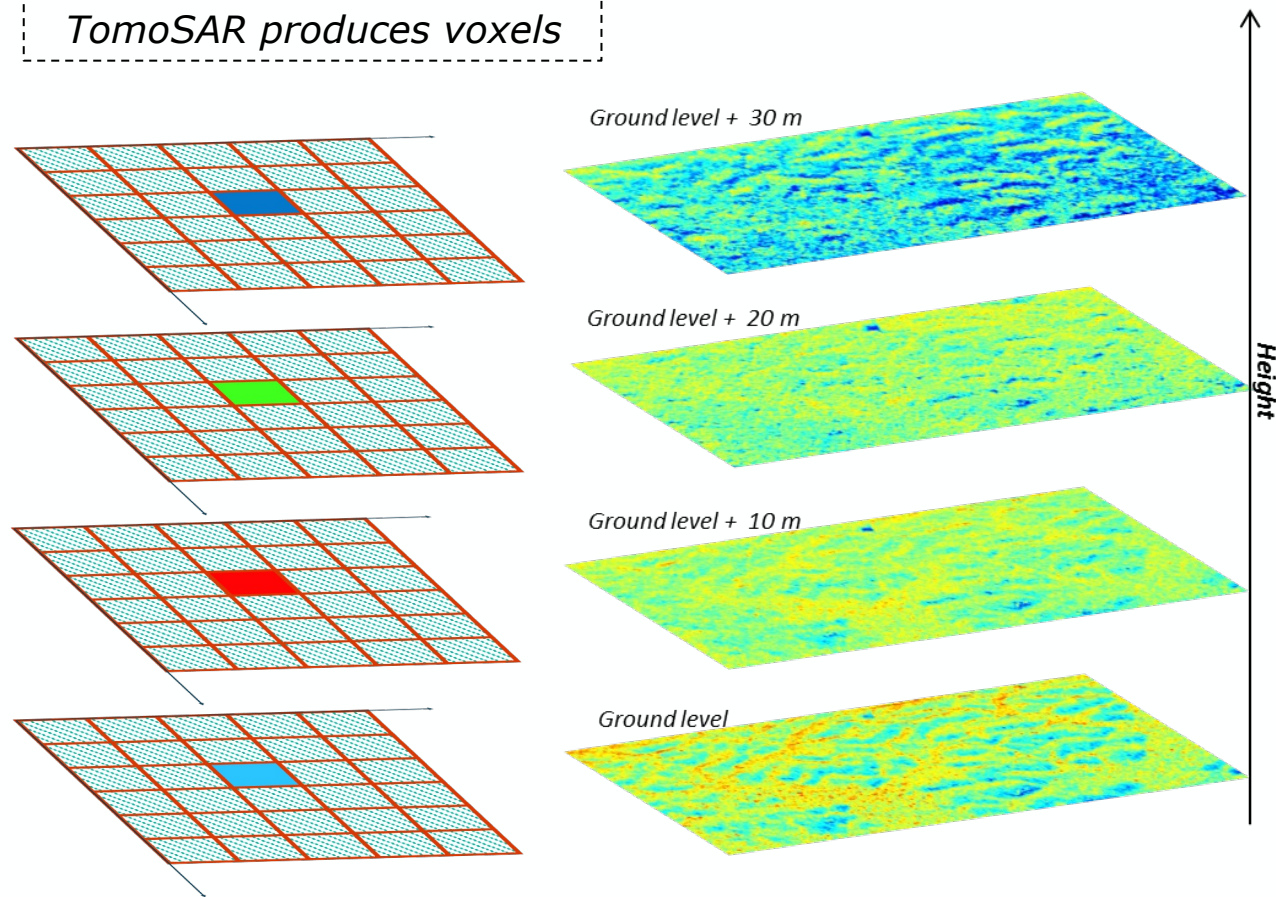
Commercial companies are demonstrating a new technological paradigm to build SAR systems

Research should commit to demonstrating new paradigms to use them

TomoSAR provides direct access to the 3D structure of Radar scattering from natural media by coherent processing of SAR data from multiple flights



*SAR produces pixels*  
*TomoSAR produces voxels*



# SAR Tomography



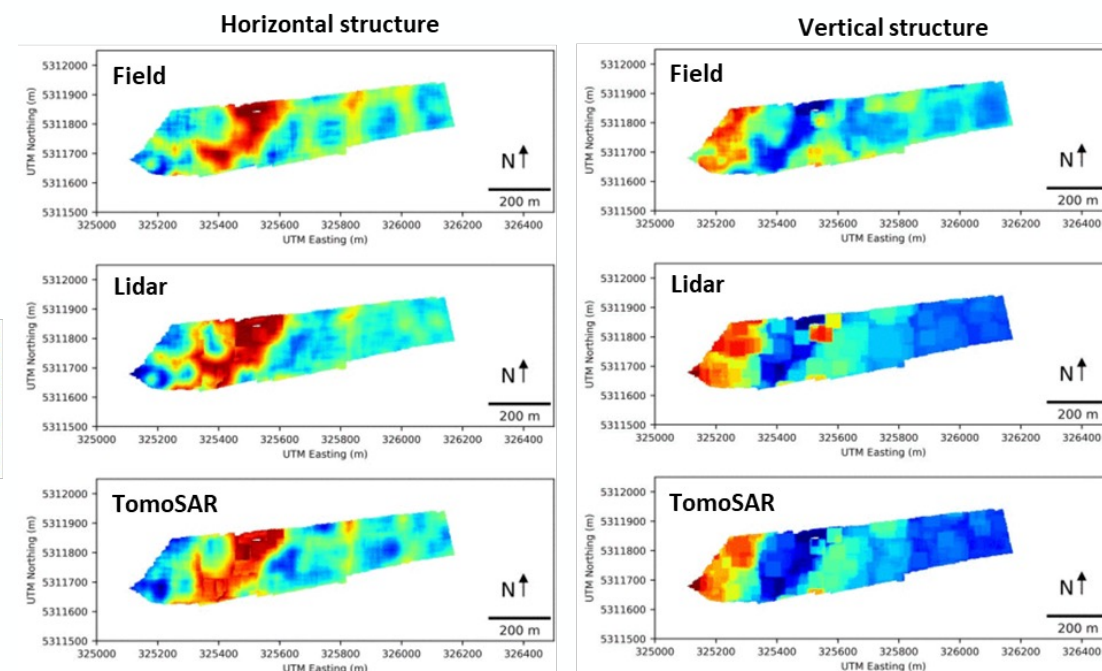
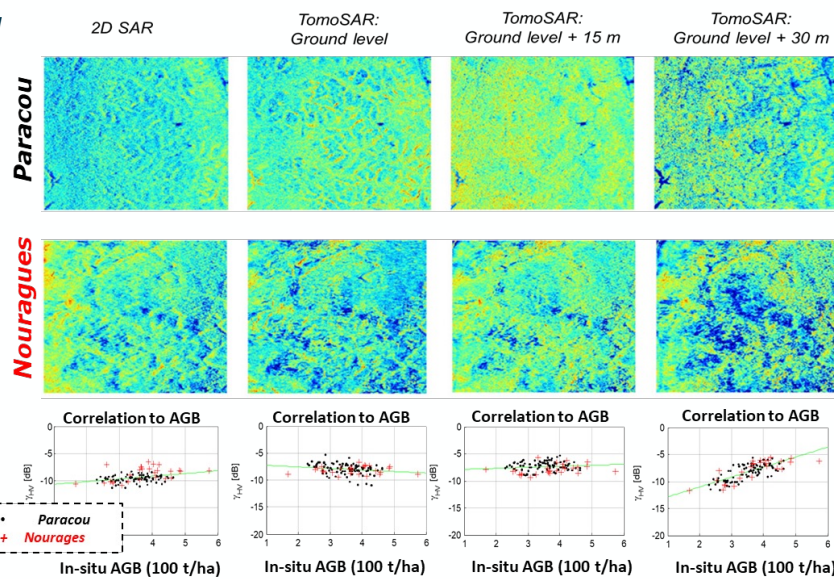
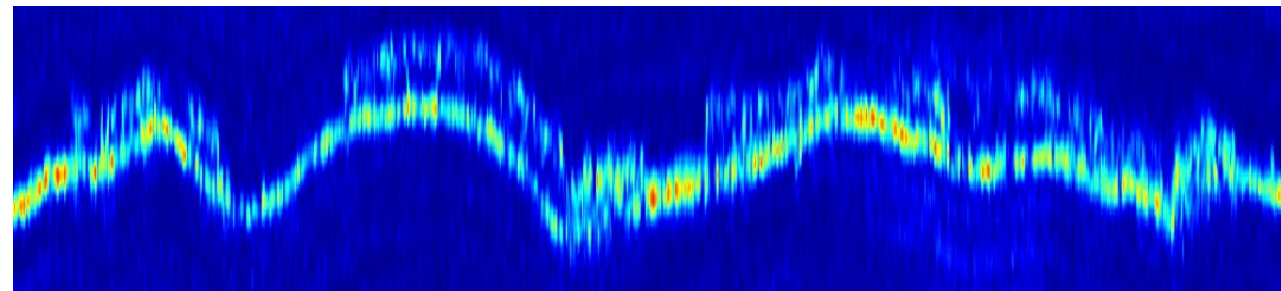
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TomoSAR provides direct access to the 3D structure of Radar scattering from natural media by coherent processing of SAR data from multiple flights

## Forested areas:

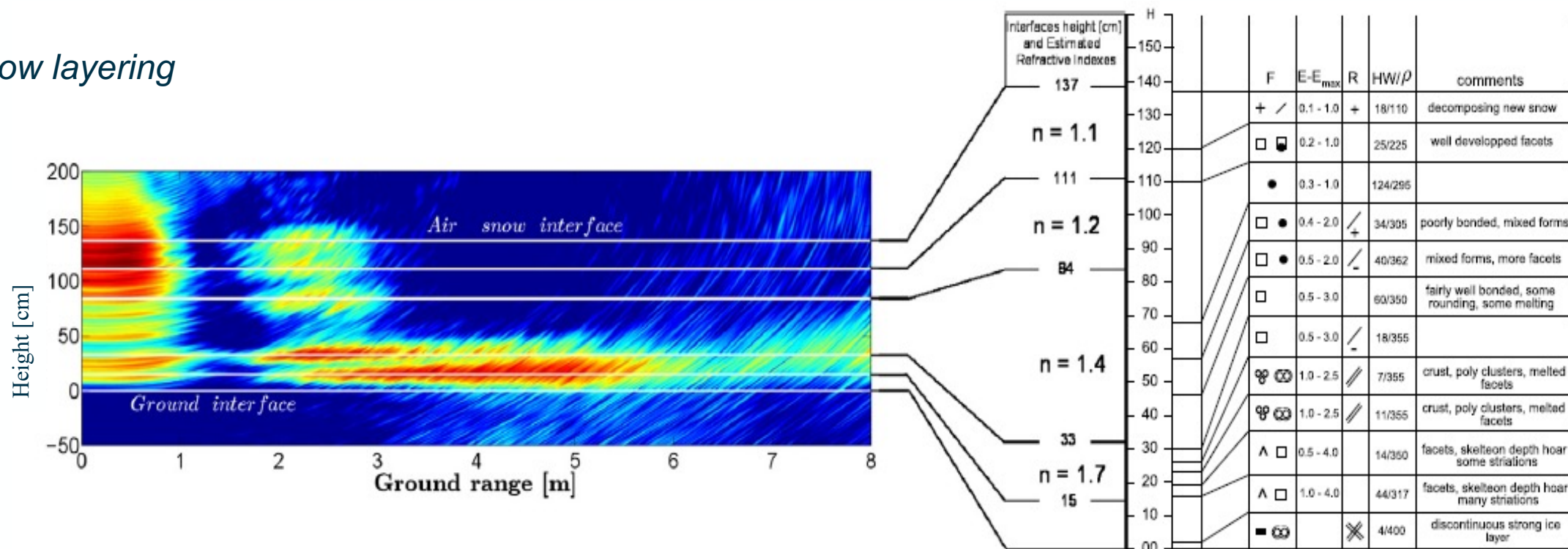
- ✓ Forest height
- ✓ True terrain elevation below the vegetation
- ✓ Improved AGB retrieval
- ✓ Vegetation structure
- ✓ BIOMASS tomographic phase tasked with global forest 3D mapping



TomoSAR provides direct access to the 3D structure of Radar scattering from natural media by coherent processing of SAR data from multiple flights

## Snow:

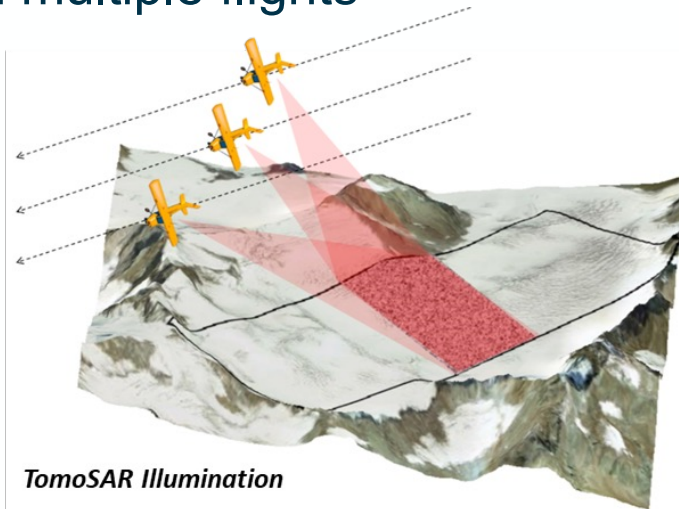
- ✓ *Snow depth*
- ✓ *Refraction index*
- ✓ *SWE*
- ✓ *Internal snow layering*



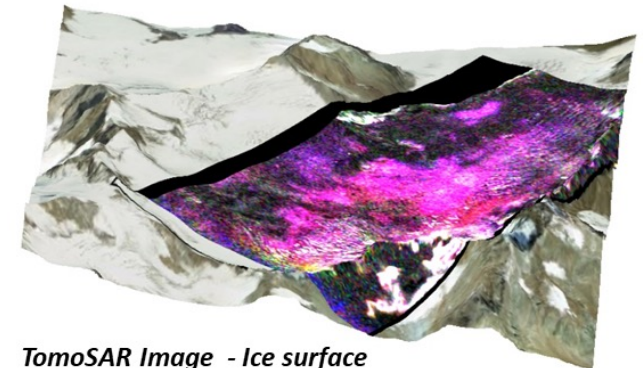
TomoSAR provides direct access to the 3D structure of Radar scattering from natural media by coherent processing of SAR data from multiple flights

## Ice and glaciers:

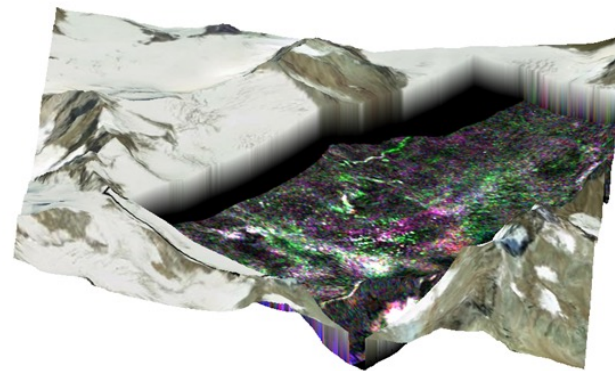
- ✓ Ice surface elevation
- ✓ Internal structure (ice/firn, crevasses, presence internal scatterers, bedrock)



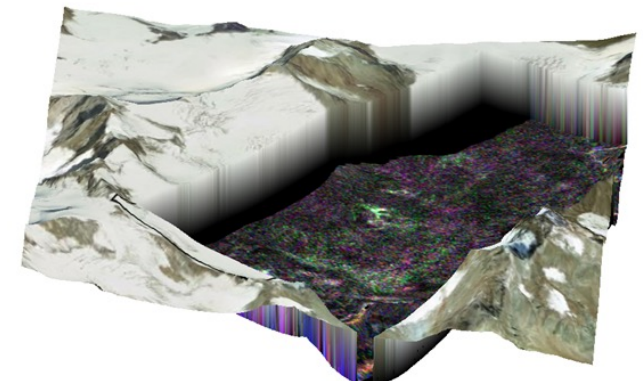
*TomoSAR Illumination*



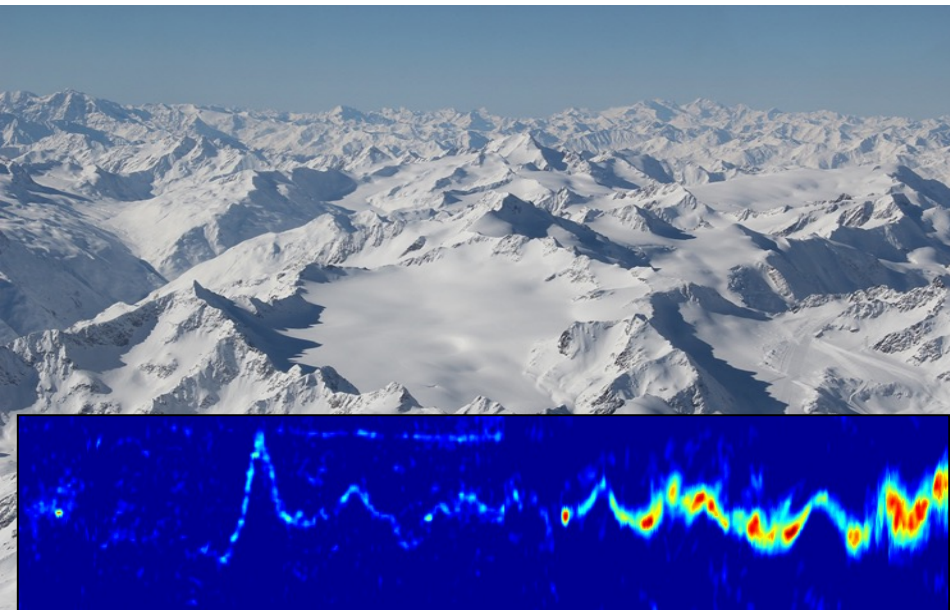
*TomoSAR Image - Ice surface*



*TomoSAR Image - 25 m below the Ice surface*



*TomoSAR Image - 50 m below the Ice surface*



Despite the many successful experimental campaigns, spaceborne tomography is yet to come for what concerns of natural scenarios.

This is largely due to the fact that that the vertical resolution provided by SAR tomography is inherently linked to the number of available view points, which - for the case of a single satellite - corresponds to the number of orbits over a given area

## ***Some examples:***

- ***Afrisar: 11 passes***
- ***TomoSense: 20 to 30 passes***
- ***AlpTomoSAR: 20 passes***
- ***AlpSAR: 36 passes***
- ***SnowLab: up to 50 passes***

Despite the many successful experimental campaigns, spaceborne tomography is yet to come for what concerns of natural scenarios.

This is largely due to the fact that the vertical resolution provided by SAR tomography is inherently linked to the number of available view points, which - for the case of a single satellite - corresponds to the number of orbits over a given area

The one mission currently appointed to gather tomographic acquisitions of forested areas is the forthcoming ESA Earth Explorer BIOMASS, for which temporal decorrelation is predicted to be successfully mitigated by the three-day revisit time and the long wavelength

Yet, vertical resolution is limited to  $\approx 23$  m



Multiple Input Multiple Output (MIMO) SAR is a most attractive solution for spaceborne tomography

*In simple terms:*

- All satellites act both as Tx and Rx
- The data acquired by a given Tx/Rx pair can be thought of as acquired by a **virtual** monostatic SAR placed in the middle (*in the limit of small bistatic angles*)

*Real formation*

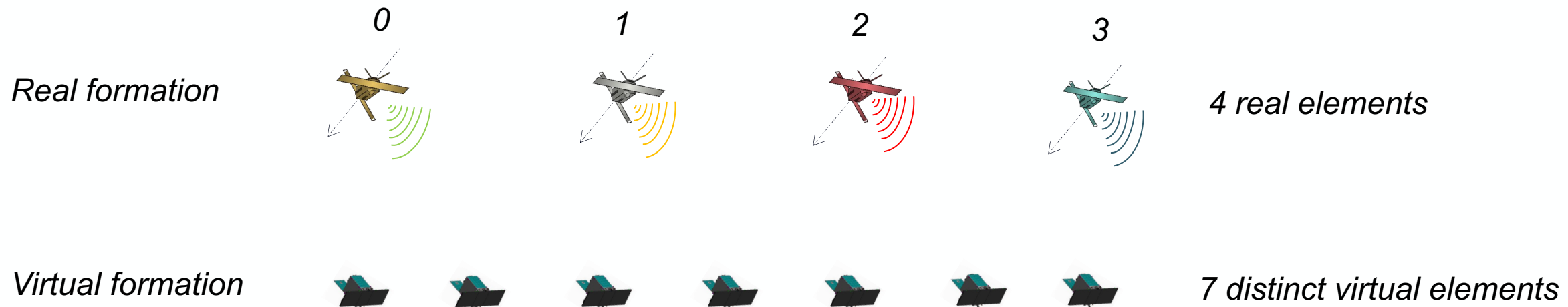


*Virtual formation*



**Key question** is how to deploy real elements to maximize the number of distinct and uniformly spaced virtual satellites

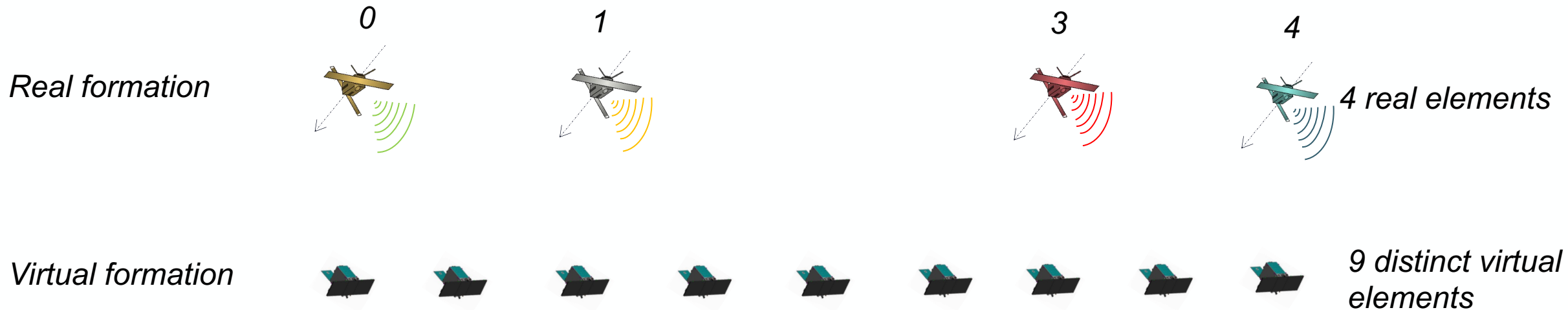
- Upper theoretical bound:  $N_v = \frac{N(N+1)}{2}$
- Approximate rules exist for sub-optimal deployment
- Optimal deployment by exhaustive search possible for few satellites



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## *Optimal deployment for a 4 satellite MIMO formation*



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- Upper theoretical bound:  $N_v = \frac{N(N+1)}{2}$
- Approximate rules exist for sub-optimal deployment
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### *Optimal deployment of 3 to 6 sats*

$N$	Optimal deployment	$N_v$
3	0 1 2	5
4	0 1 3 4	9
5	0 1 3 5 6	13
6	0 1 3 5 7 8	17

With 4 (5) optimally deployed physical satellites the number of virtual sensors is just 9 (13)

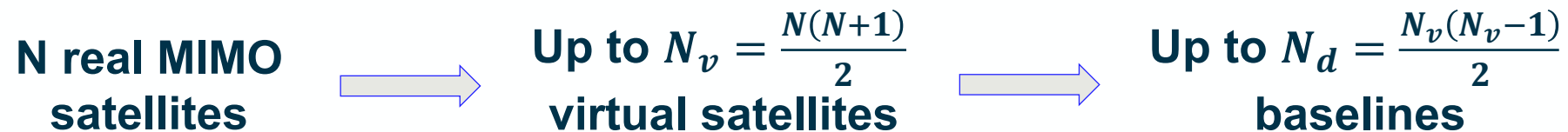
Still far from state-of-the-art tomographic campaigns.

**Correlation tomography** is an alternative approach to tomographic imaging that proceeds from the set of InSAR coherences gathered at different baselines

- Proposed in the context of SAR by *Treuhft* (2000) and *Claude* (2007)
- Largely considered in the context of bistatic mission proposals like SAOCOM-CS and PARSIFAL

Assuming correlation tomography, tomographic imaging depends on the set of available baselines, i.e.: the set of the **differences** in sensor positions

Keeping mind that sensors can be *virtual*, it is immediate to see the benefit:



The number of available baselines ideally goes with up to the *4-th* power of the number of physical satellites

# Minimum Redundancy Virtual Array



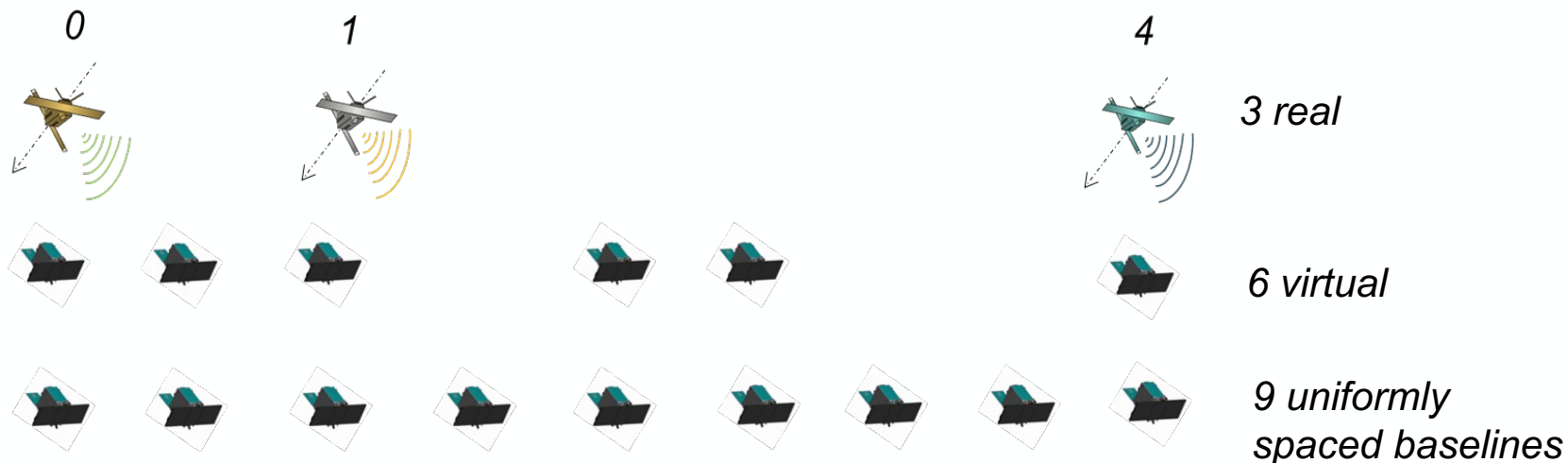
Optimal sensor deployment can then be cast in the following terms:

*find the positions of  $N$  physical Rx/Tx sensors such that the set of position differences among all resulting virtual sensors form a uniformly-spaced array of maximum length.*

Well known concept in array literature

Optimal deployment is referred to as Minimum Redundancy Virtual Array (MRVA)

$N$	Optimal deployment	$N_d$
3	0 1 4	9
4	0 1 9 12	25
5	0 1 14 19 22	45
6	0 1 9 10 34 37	75



## *Two problems*

**Channel access:** a tacit assumption in MIMO literature is that the echoes associated with different transmissions can be perfectly separated at the Rx. To do that, transmission of orthogonal waveforms is generally assumed.

This is simply impossible for the case of SAR\*, unless:

- Waveform design is coupled with beamforming in elevation at Rx
- The transmissions occur in separated frequency bands

*\*G. Krieger, MIMO-SAR, Opportunity and Pitfalls, TGRS, 2014.*

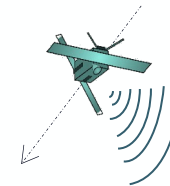
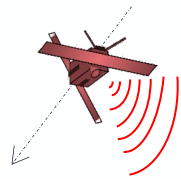
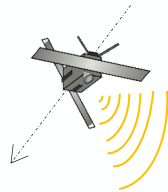
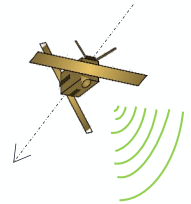
**Spatial decorrelation:** array literature assumes always isotropic point scatterers. No consideration is present about target decorrelation as a function of baseline

⇒ The concept of MRVA is not directly applicable to the case of TomoSAR

## Two solutions

**Channel access:** assume Frequency Division Multiplexing (FDM)

- All satellites transmit simultaneously on different frequency bands and receive all transmitted bands
- Guided by the idea to enable an increase of the number of satellites by simplifying individual elements

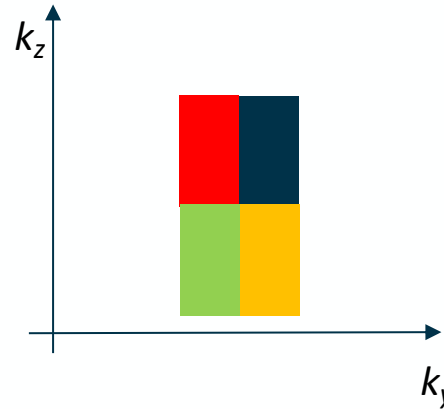
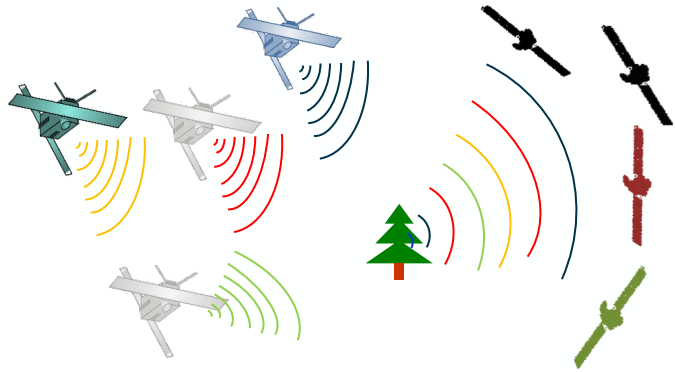


**Spatial decorrelation:** design the formation to optimize resolution while accounting for which spatial wavenumbers are captured to ensure spectral overlap



## Diffraction Tomography\*

- The information retrieved about an illuminated weak-scattering object corresponds to a specific region in the wavenumber domain, depending on wavelength, bandwidth, Tx and Rx position

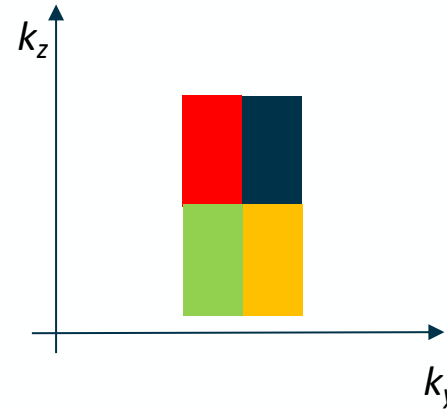
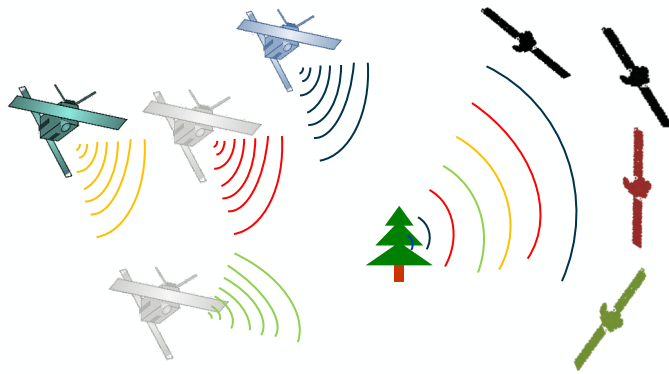


- In a multistatic experiment, different regions can be combined to form a bigger one by properly positioning different Tx/Rx pairs
- Largely used in crystallography and seismic exploration

*\*Diffraction tomography and multisource holography applied to seismic imaging, Wu and Toksoz, GEOPHYSICS, VOL. 52, NO. 1 (JANUARY 1987);*

## Diffraction Tomography\*

- The information retrieved about an illuminated weak-scattering object corresponds to a specific region in the wavenumber domain, depending on wavelength, bandwidth,  $T_x$  and  $R_x$  position



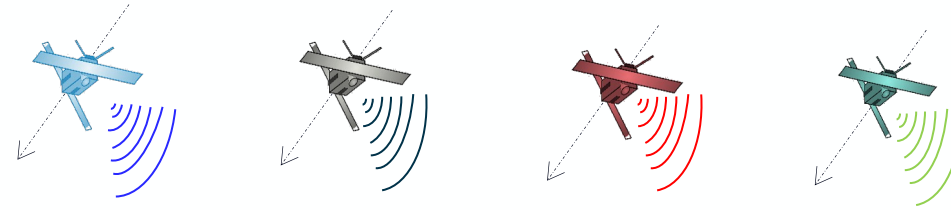
- In a multistatic experiment, different regions can be combined to form a bigger one by properly positioning different Tx/Rx pairs
- ***Kz is here the vertical wavenumber. Don't confuse it for the interferometric wavenumber!***

*\*Diffraction tomography and multisource holography applied to seismic imaging, Wu and Toksoz, GEOPHYSICS, VOL. 52, NO. 1 (JANUARY 1987);*

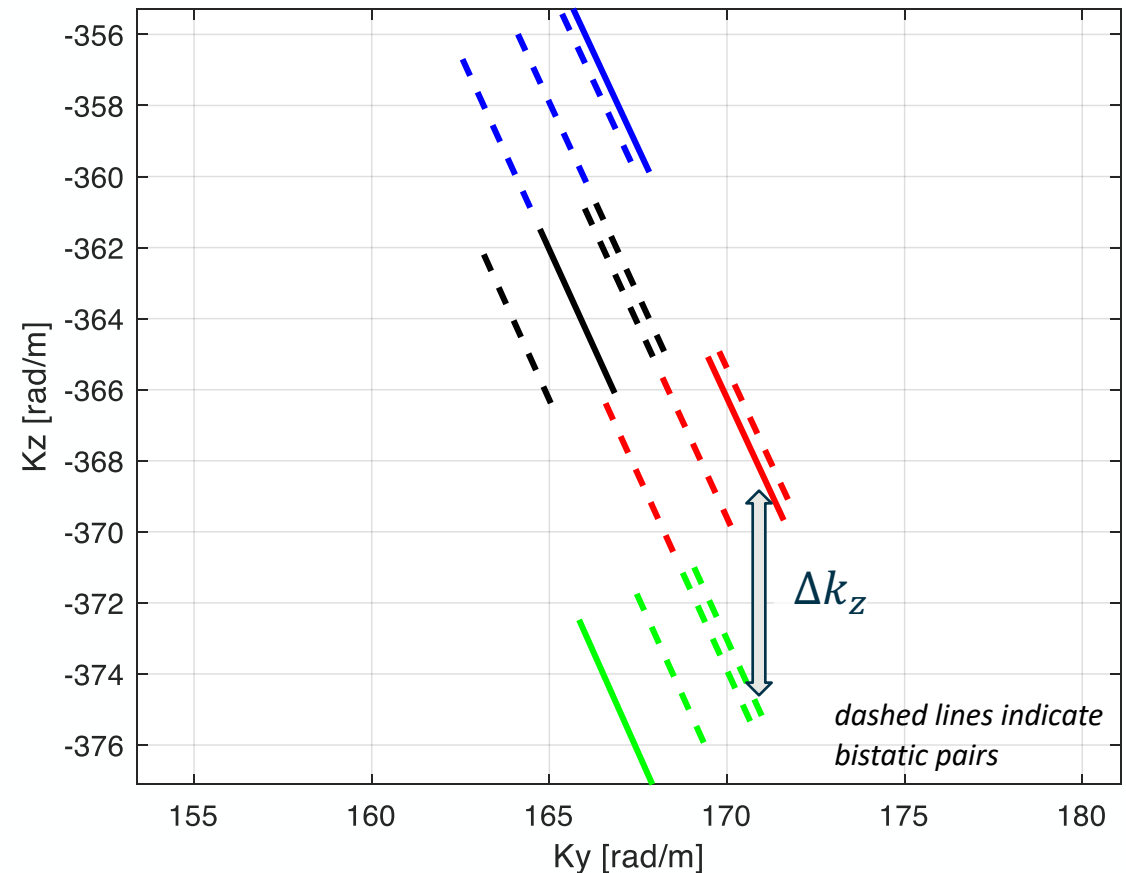
Mapping from  $(f, \theta)$  to  $(k_y, k_z)$  is given as:

$$k_y = \frac{2\pi f_{Tx}}{c} (\sin(\theta_{Rx}) + \sin(\theta_{Tx}))$$

$$k_z = -\frac{2\pi f_{Tx}}{c} (\cos(\theta_{Rx}) + \cos(\theta_{Tx}))$$



Spectral coverage: same color = same Tx

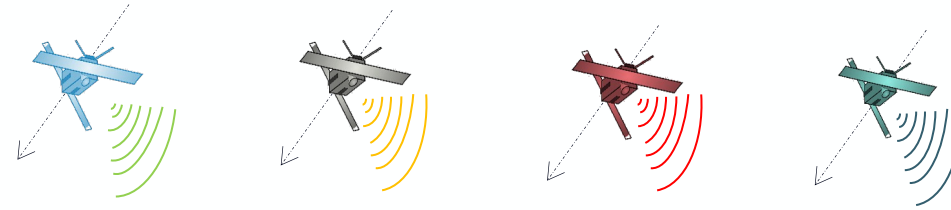


- Each Tx/Rx SAR acquisition is represented by a segment in the  $(k_y, k_z)$  plane
- Spatial resolution is the reciprocal of the extent of wavenumber coverage
- Spatial ambiguity is given by spacing between segments
- The interferometric wavenumber between any two acquisitions is obtained as the vertical separation between any two segments

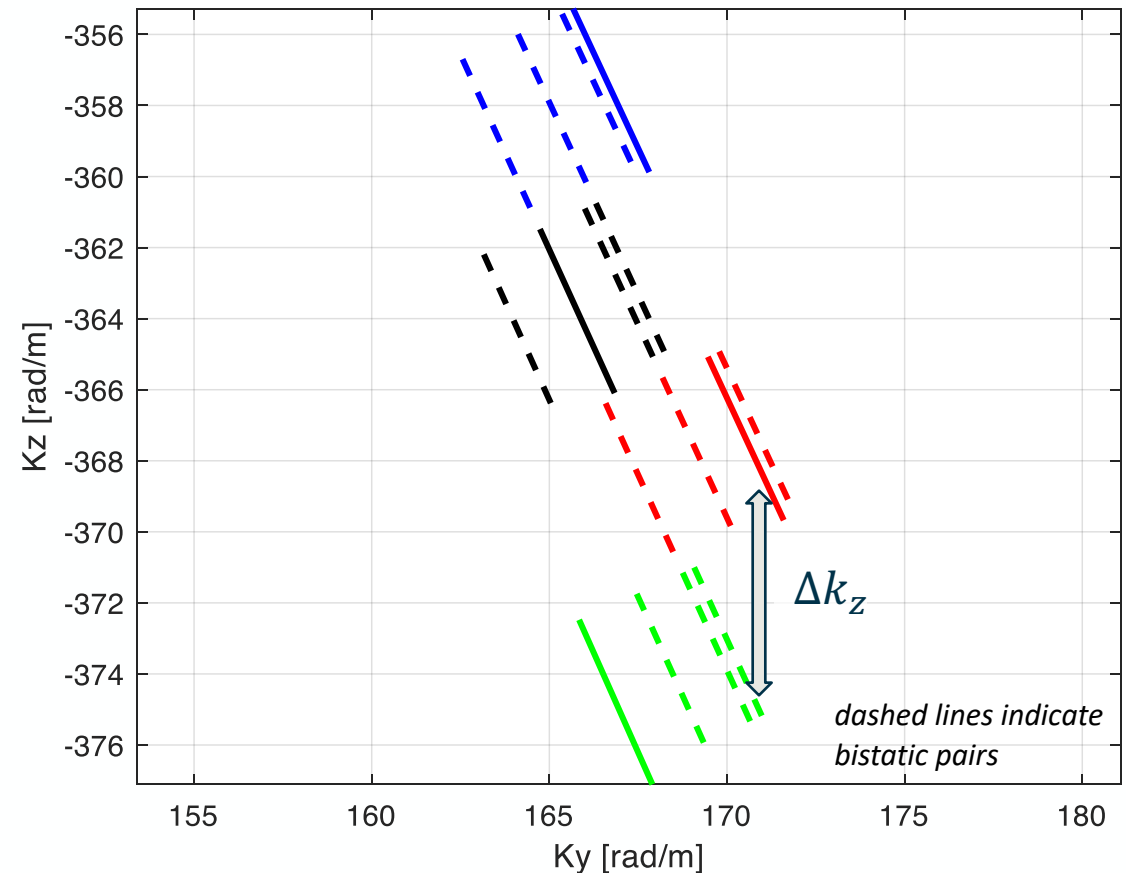
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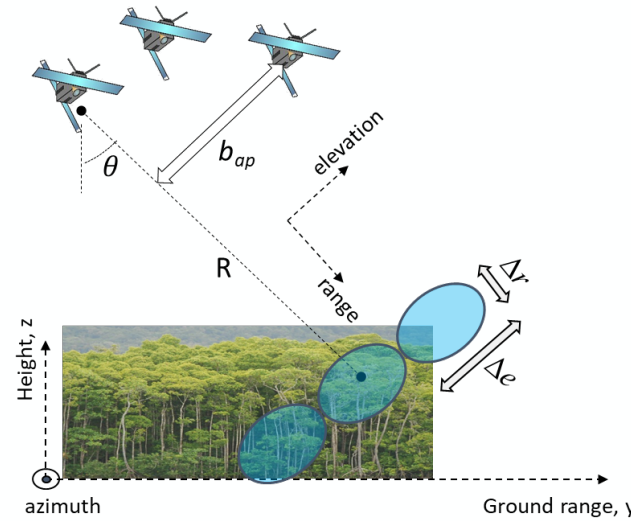
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*this is what is conventionally called  $k_z$  but we call it here  $\Delta k_z$  to avoid confusion*

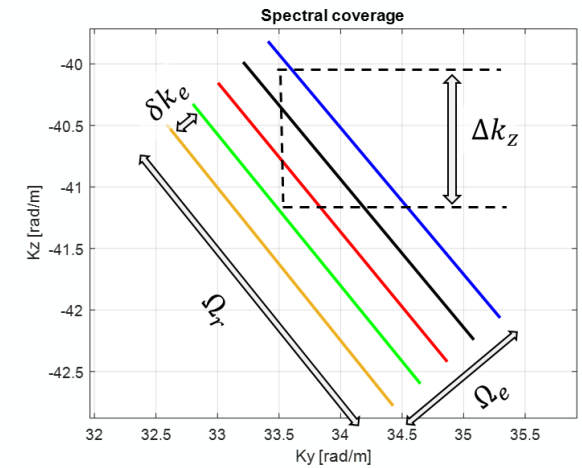
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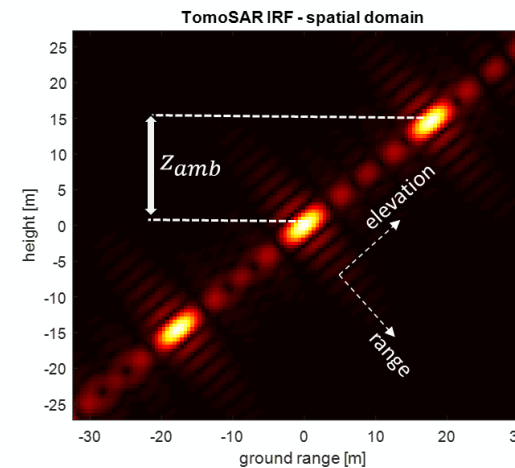
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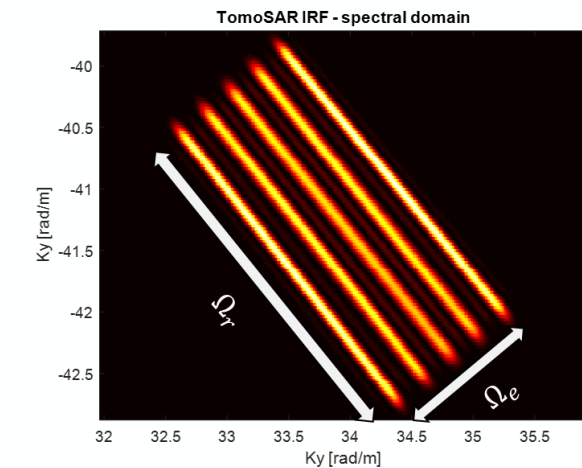
(a)



(b)



(c)



(d)

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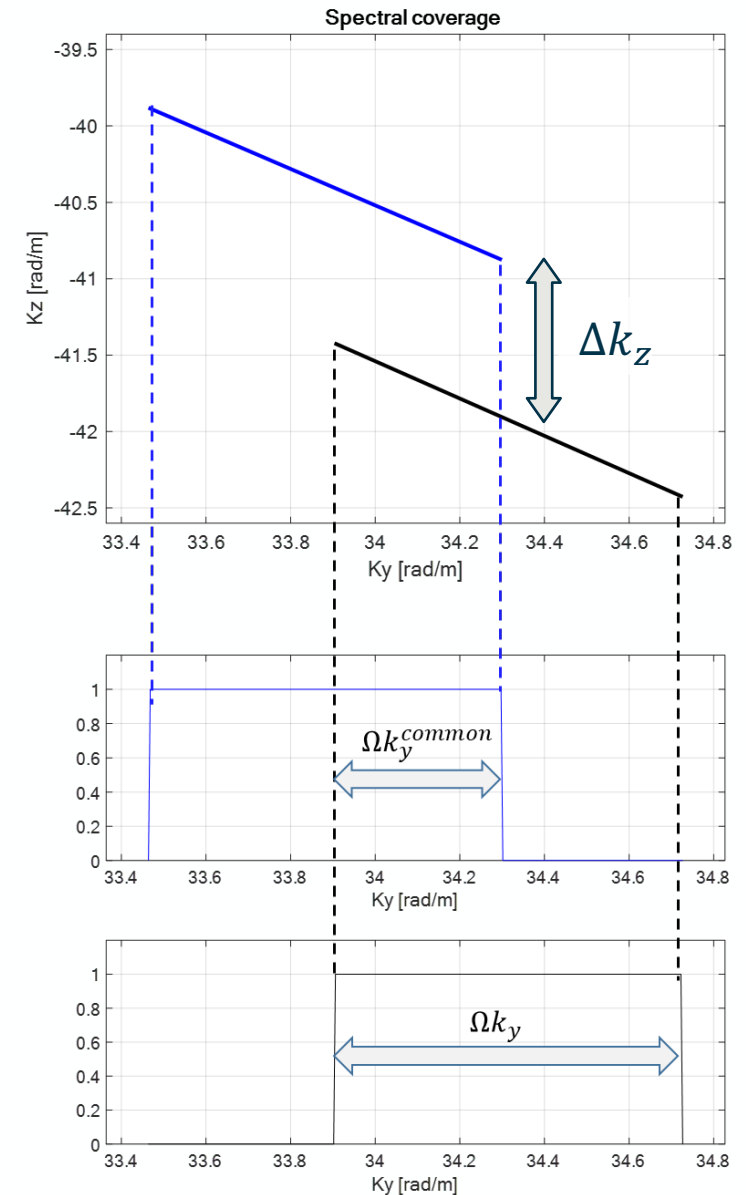
Spatial decorrelation in any two images is accounted for by taking the projection of individual wavenumber segments onto the  $k_y$  axis

⇒ straightforward evaluation of the interferometric coherence magnitude based on a visual analysis of the overlapping regions between the projections of any two segments

$$\gamma = \frac{\Omega k_y^{common}}{\Omega k_y}$$

On this ground, optimal deployment of a formation of N MIMO Rx/Tx sensors can be discussed in a graphical manner.

- Interferometric pairs are marked as **valid** depending on the overlap between their projections onto the  $k_y$  axis.
- For valid pairs, the interferometric wavenumber is obtained as the vertical separation between the associated segments



The resulting optimization principle is referred to as **Minimum Redundancy Wavenumber Illumination**



*find the positions of  $N$  physical Rx/Tx sensors such that the set of **valid** vertical separations among all illuminated wavenumbers is as close as possible to forming a uniformly spaced array of maximum length.*

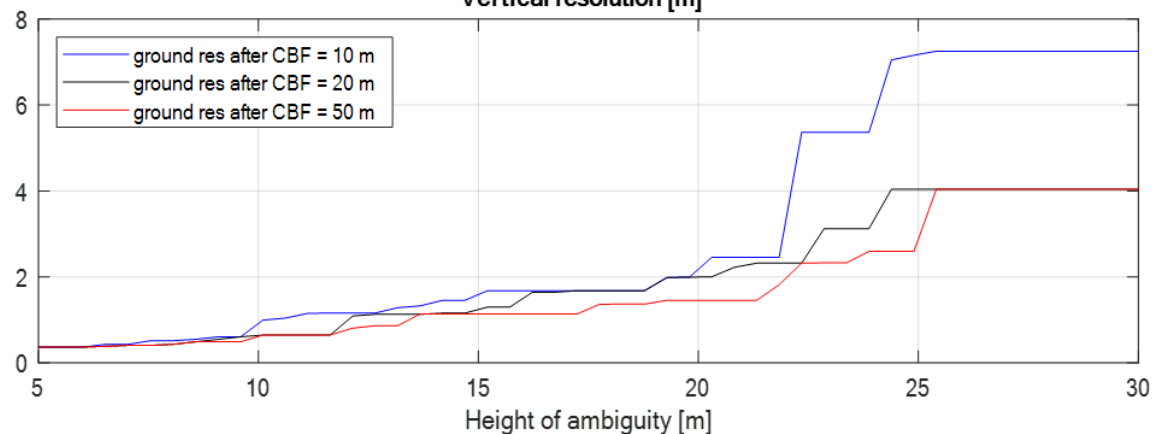


Optimization is carried out via exhaustive search:

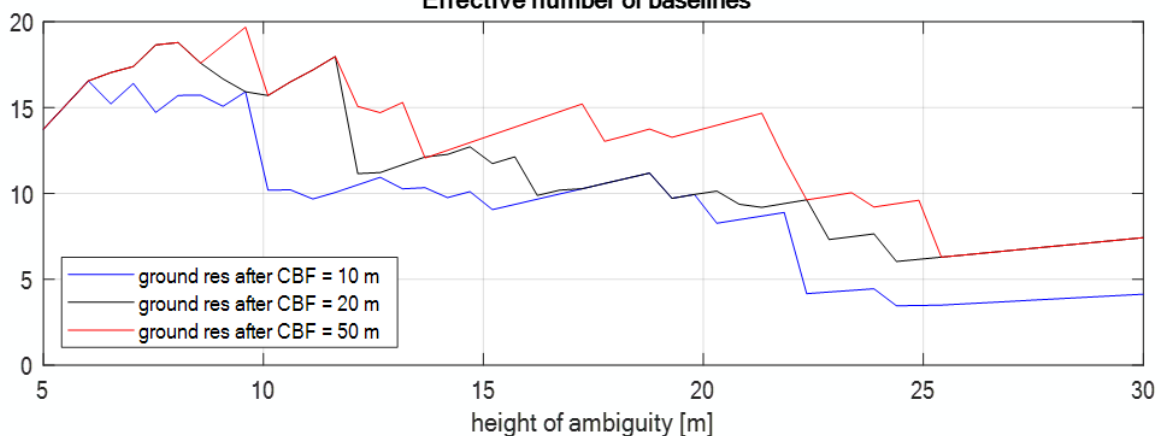
- Start with perfect alignment along  $k_y$
- Explore all possible combinations of satellite positions
- For every possible combination, annotate:
  - ✓ Resulting vertical resolution
  - ✓ Height of ambiguity
  - ✓ Spectral overlap, quantified as the resulting ground resolution after Common Band Filtering is performed

## 4 satellites

Vertical resolution [m]

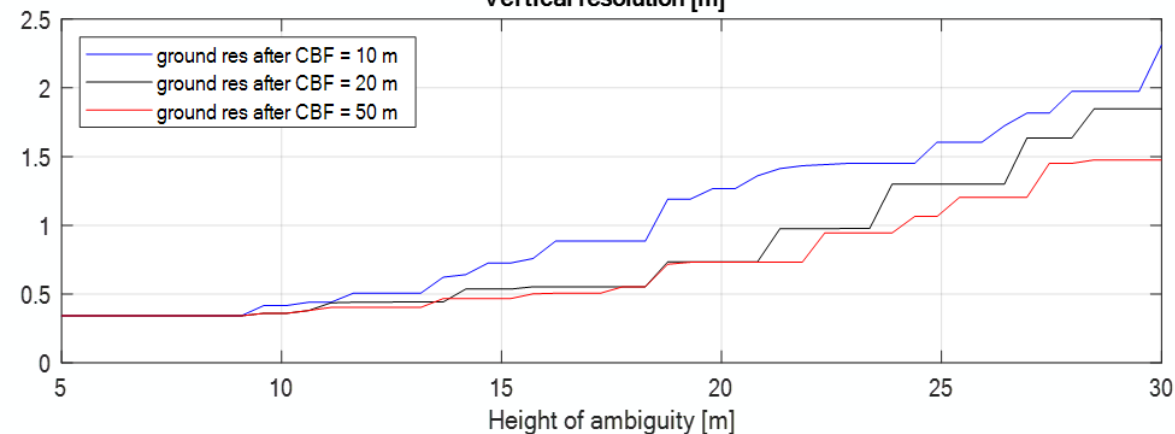


Effective number of baselines

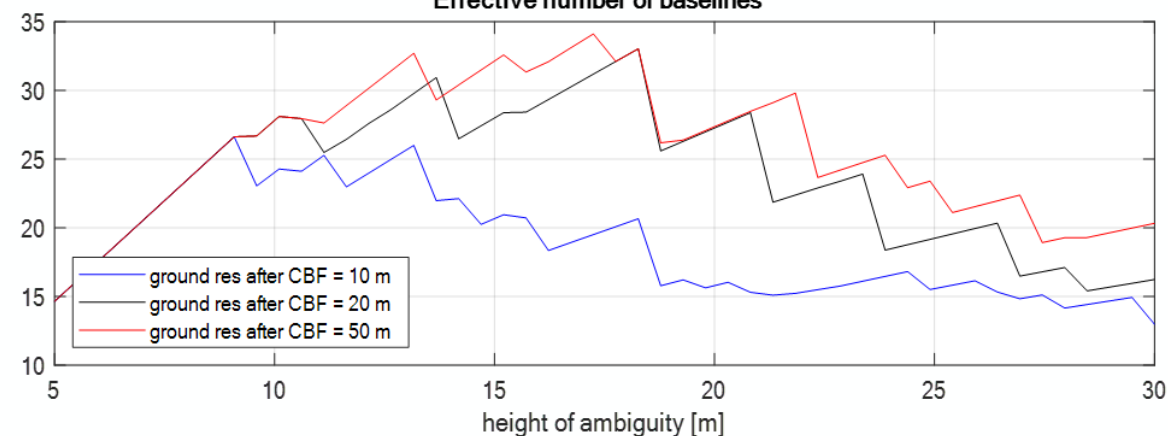


## 5 satellites

Vertical resolution [m]



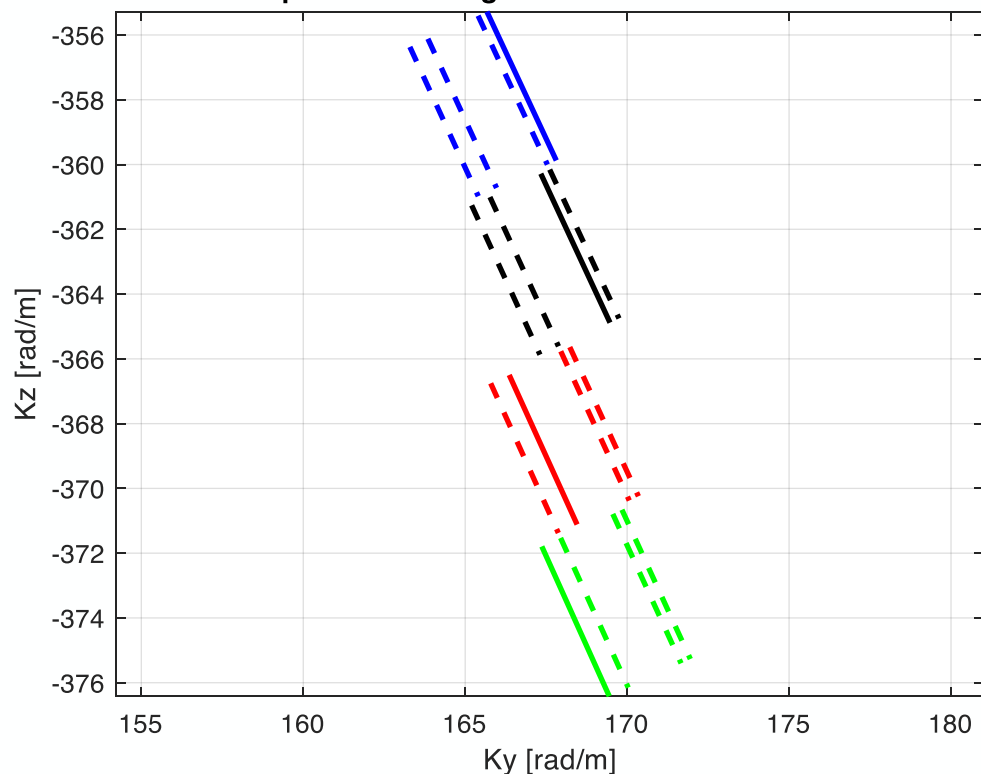
Effective number of baselines





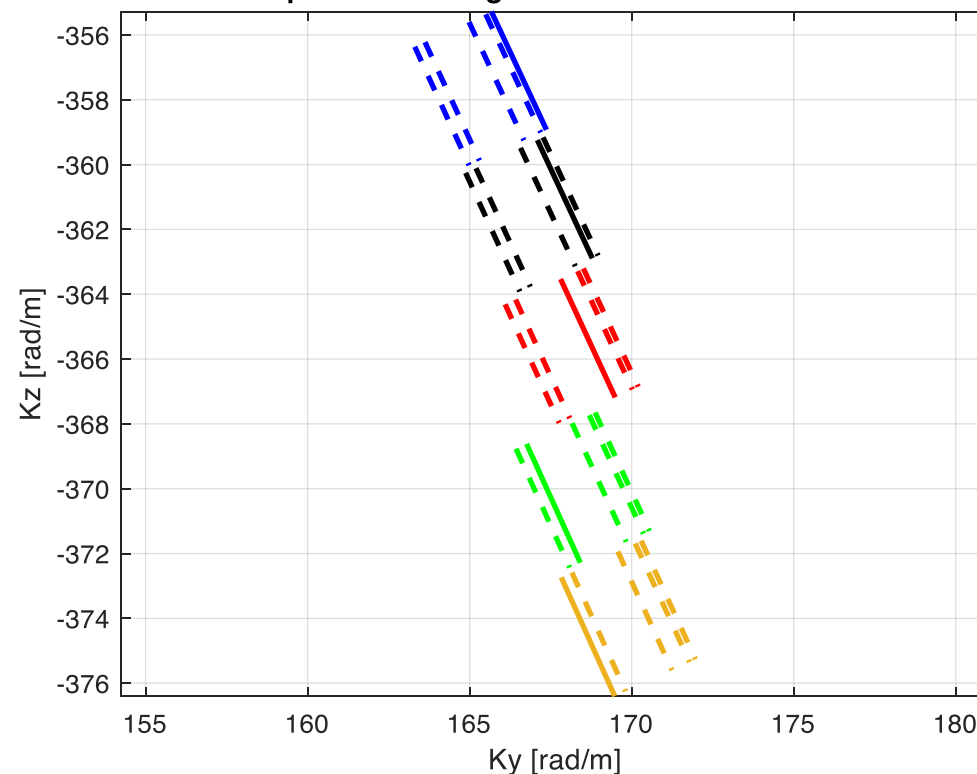
## 4 satellites

Spectral coverage: same color = same Tx



## 5 satellites

Spectral coverage: same color = same Tx



$N$	Worse ground resolution after CBF (m)	Optimal deployment ( $^{\circ}$ )				$Z_{amb}$ (m)	$\delta z$ (m)	$N_{eff}$	
4	10	25.00	24.91	24.41	24.24	8.4	0.51	16	
5	10	25.00	24.93	24.64	24.22	24.08	10.5	0.41	25

Geometrical simulation by projection of random scatterers based on bistatic delays

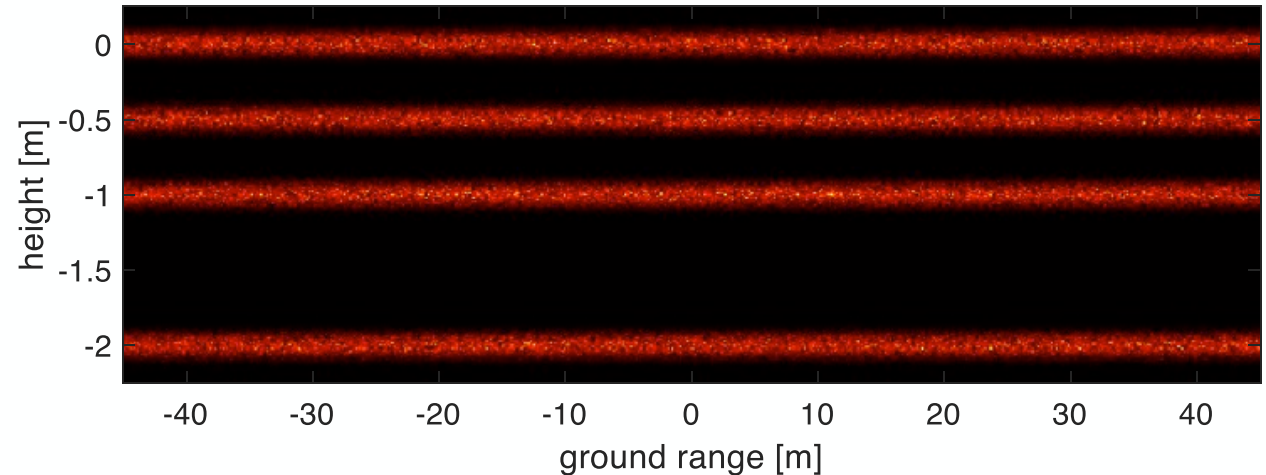
⇒ Spatial decorrelation is automatically accounted for

Simulated scenario accounts for 4 closely spaced layers

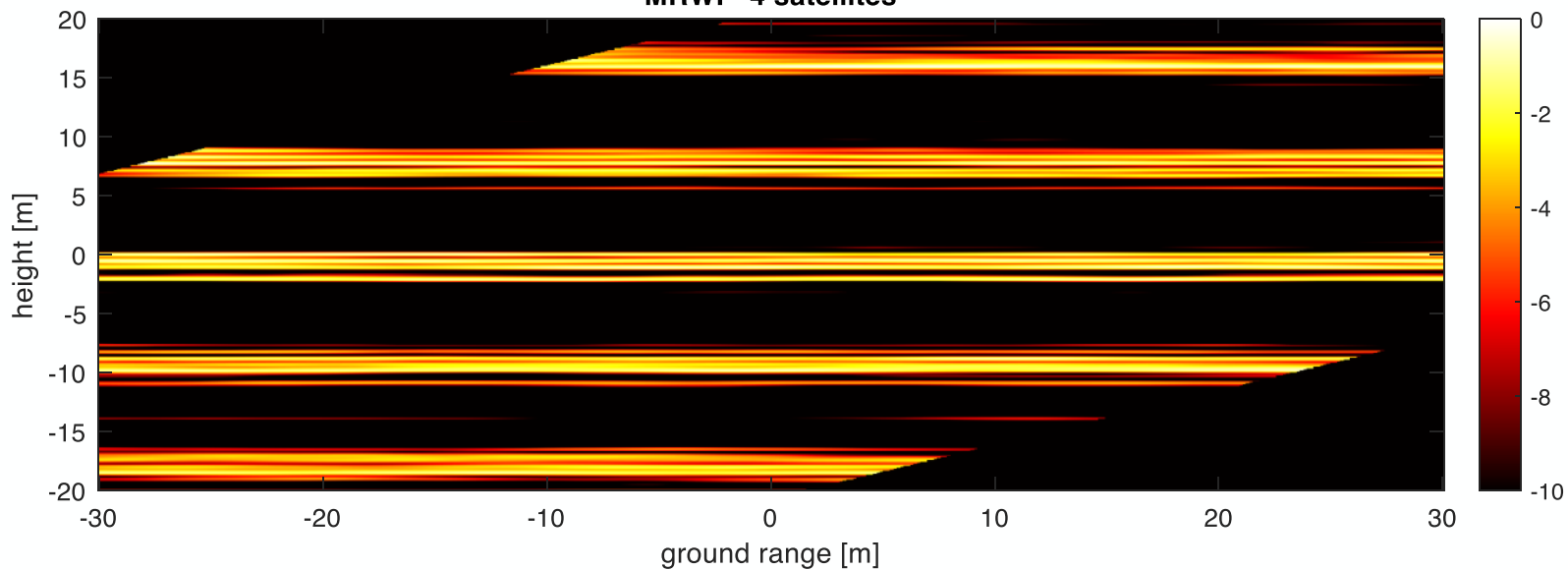
## Simulation parameters

- $N = 4$  or 5 satellites
- Formation deployment according to MRVA and MRWI
- Total bandwidth 500 MHz (split among the  $N$  satellites)
- Guard band of 6 MHz
- Reference incidence angle  $\theta = 25^\circ$

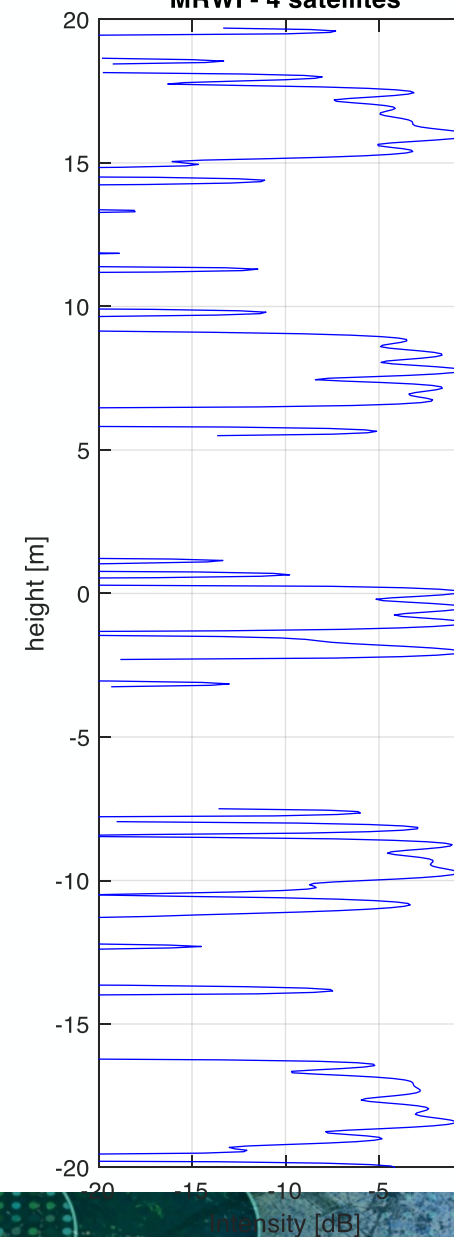
Simulated scenario



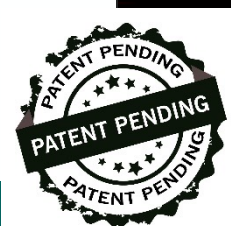
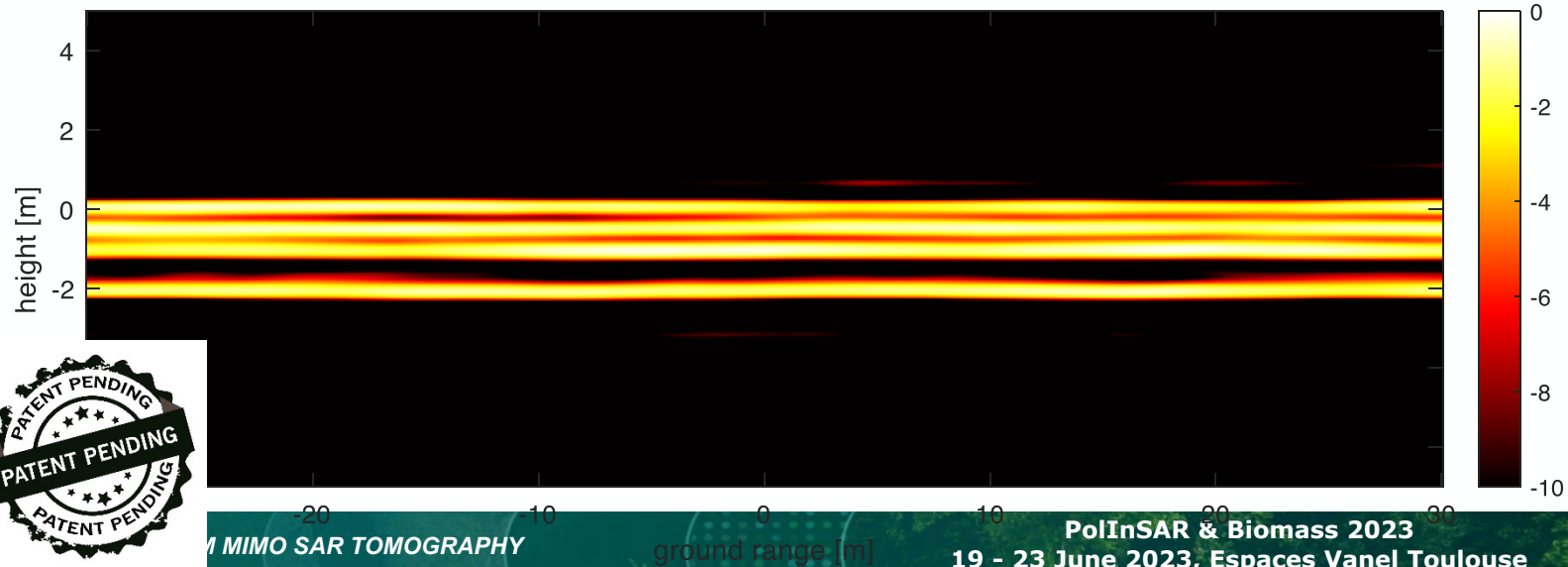
MRWI - 4 satellites



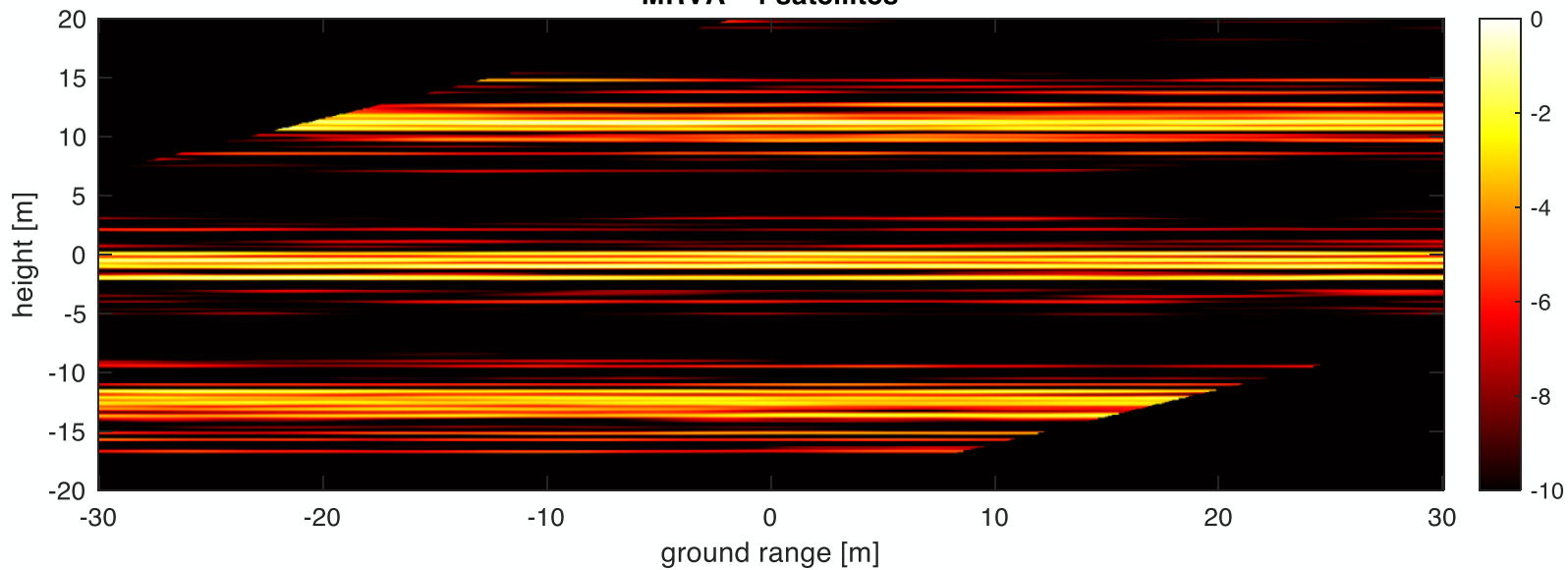
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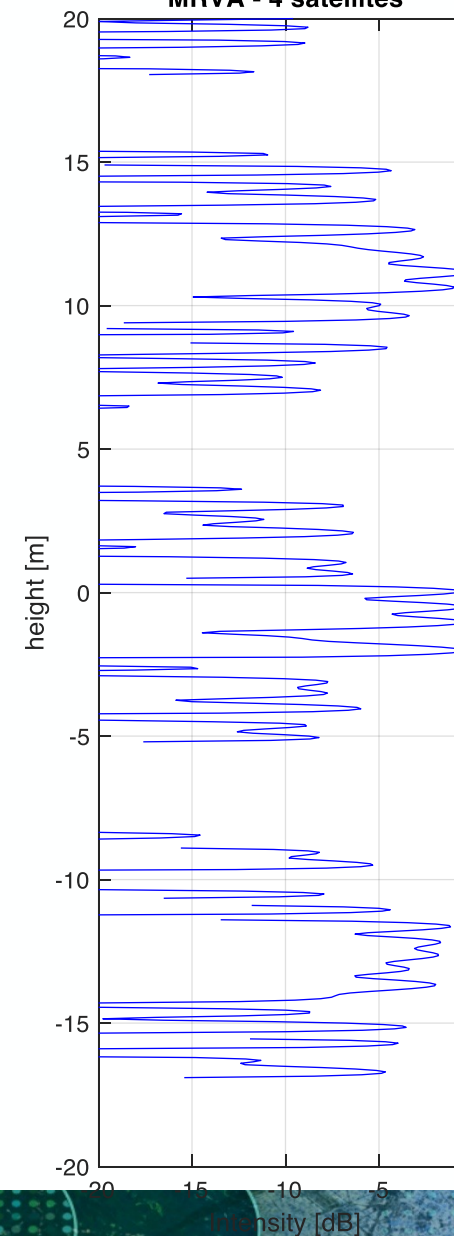
MRWI - 4 satellites



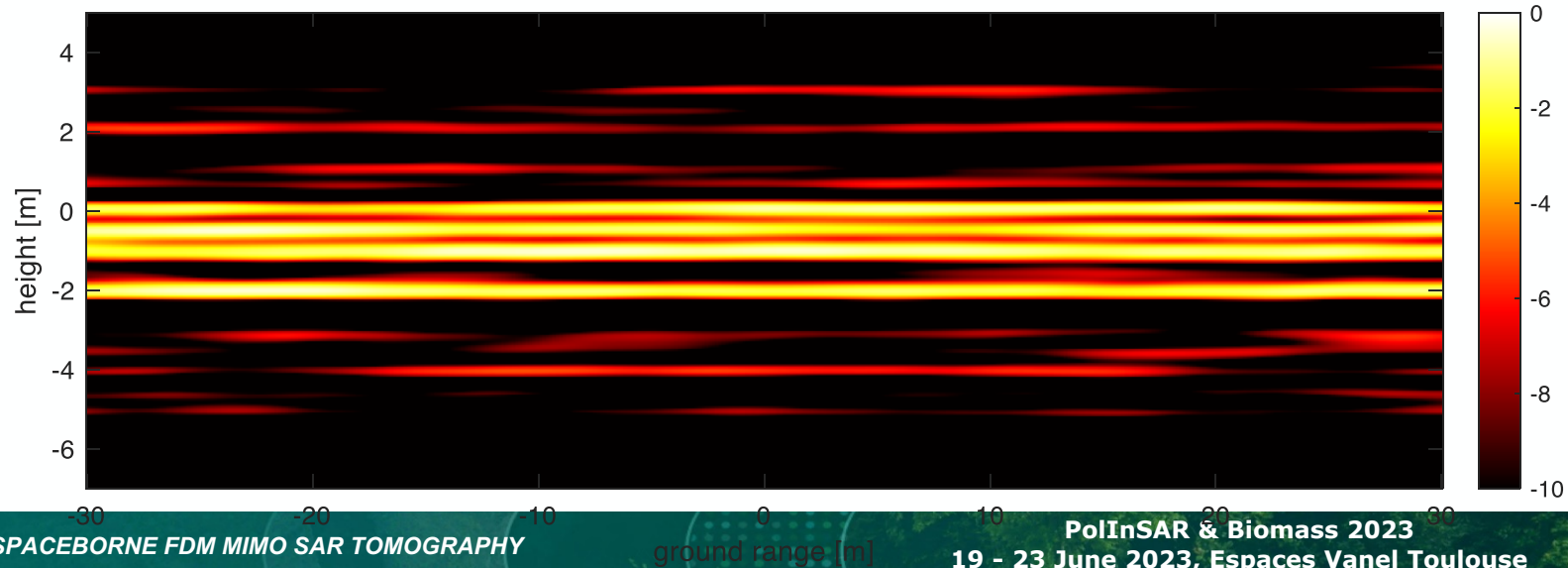
MRVA - 4 satellites



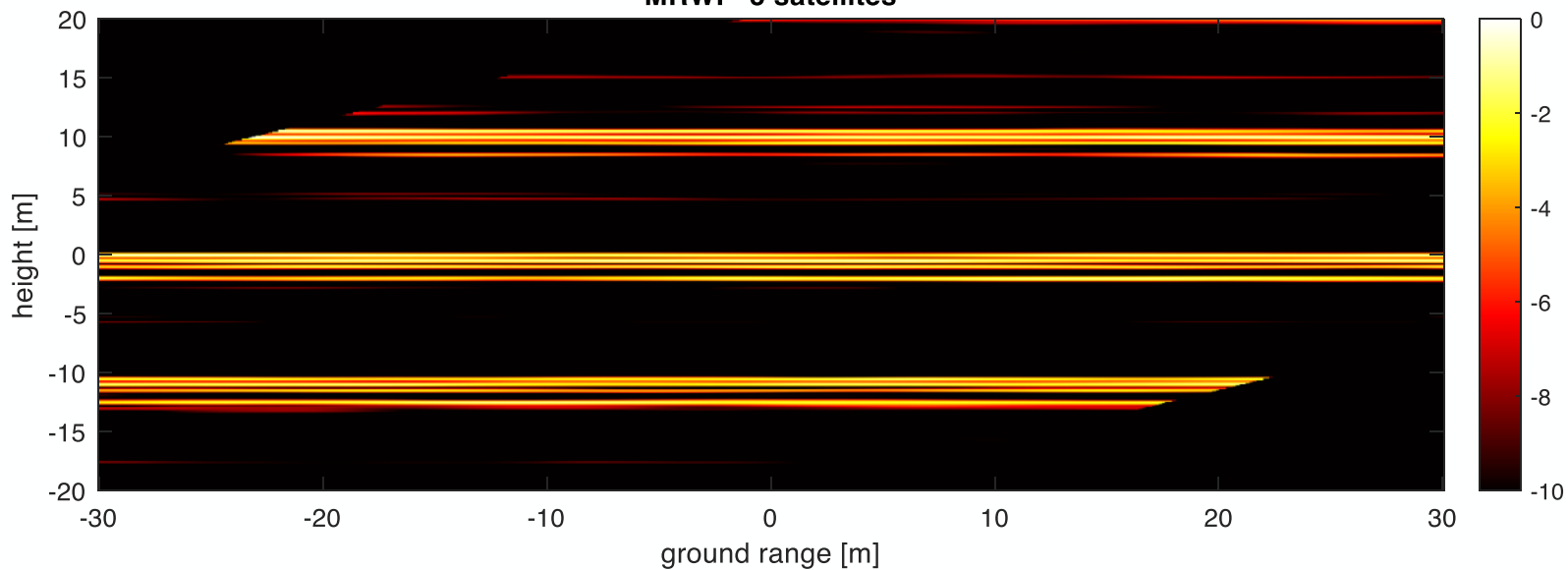
MRVA - 4 satellites



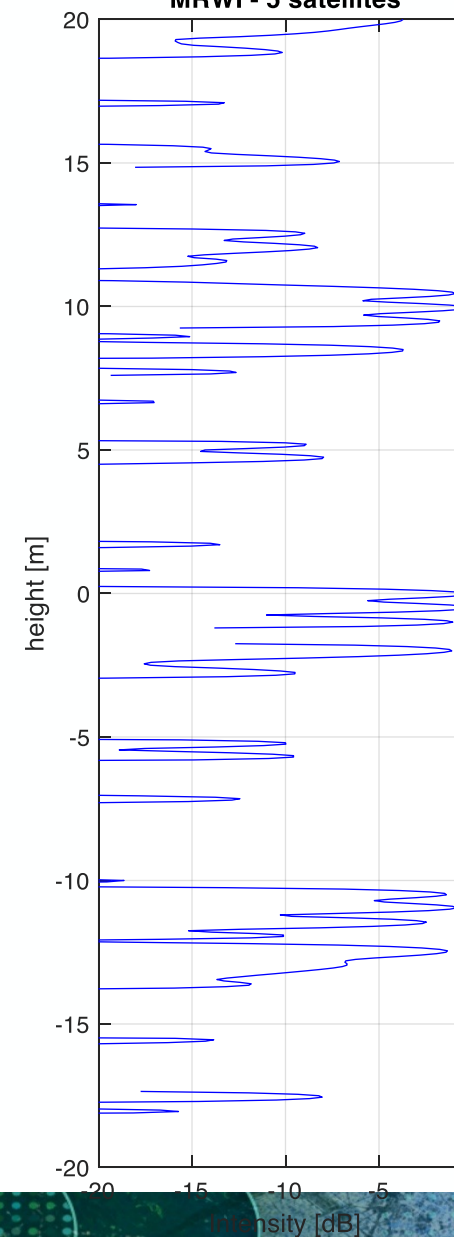
MRVA - 4 satellites



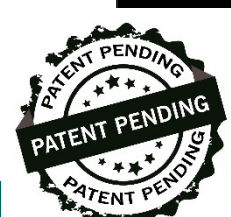
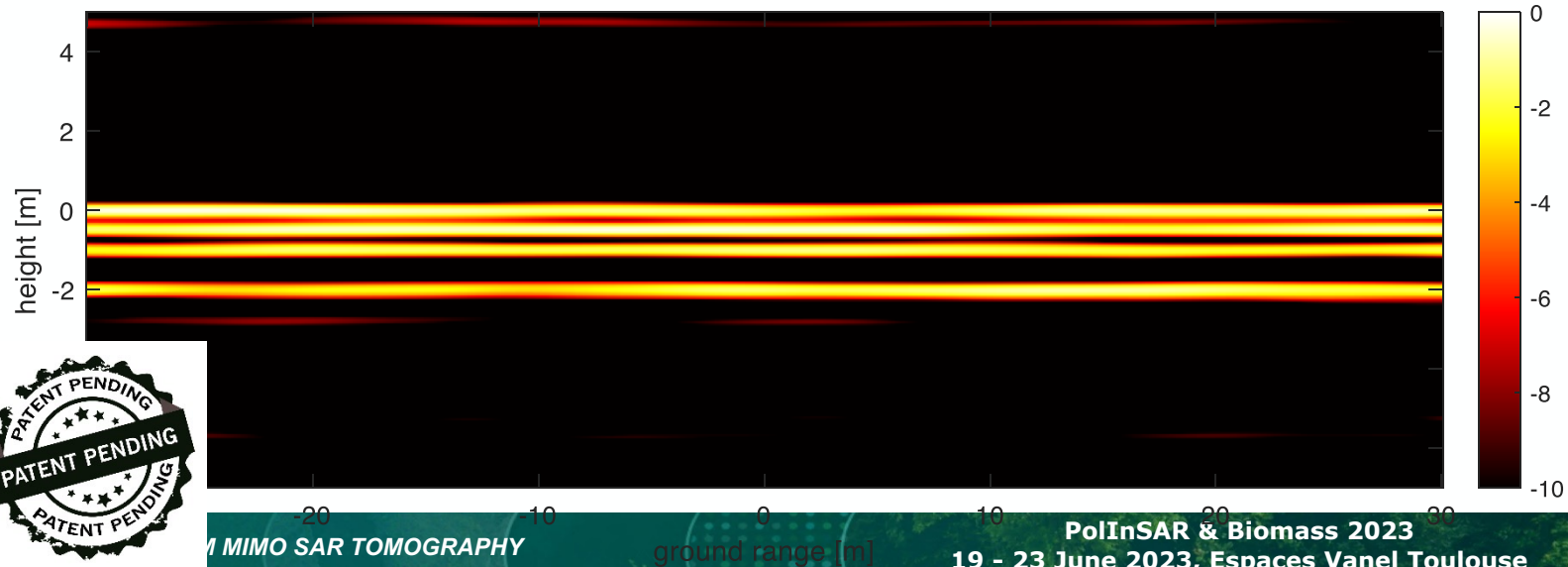
MRWI - 5 satellites



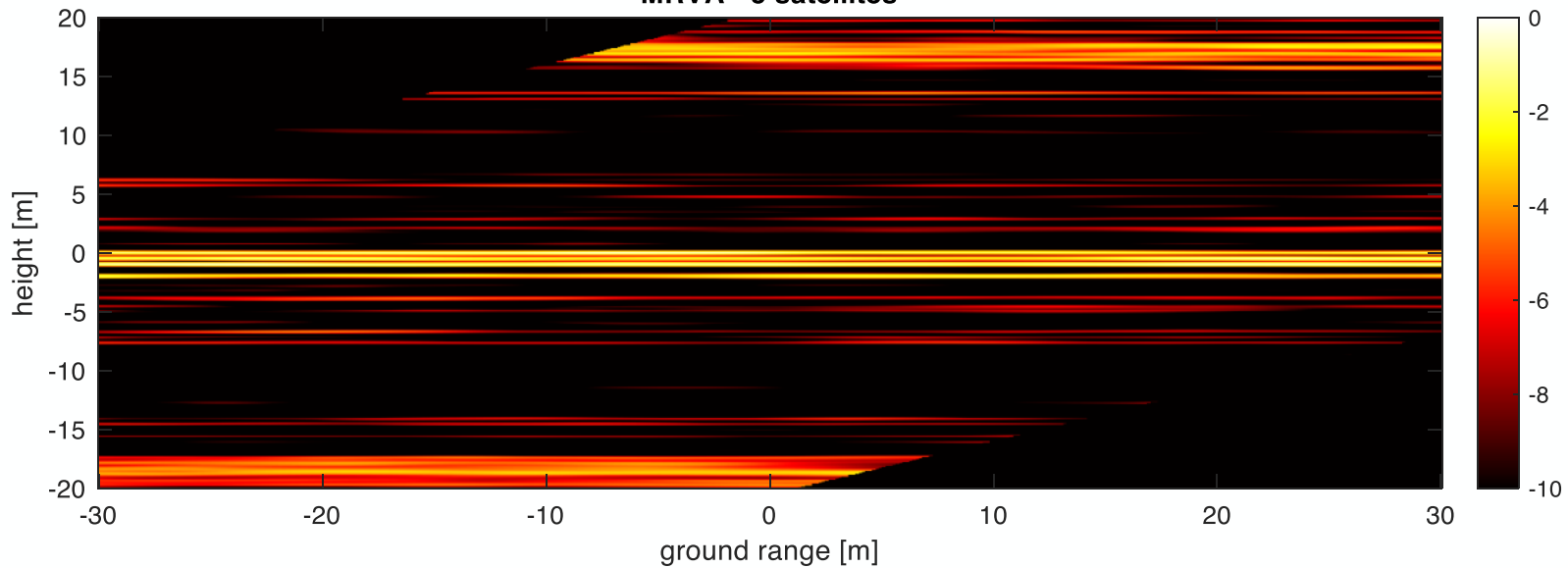
MRWI - 5 satellites



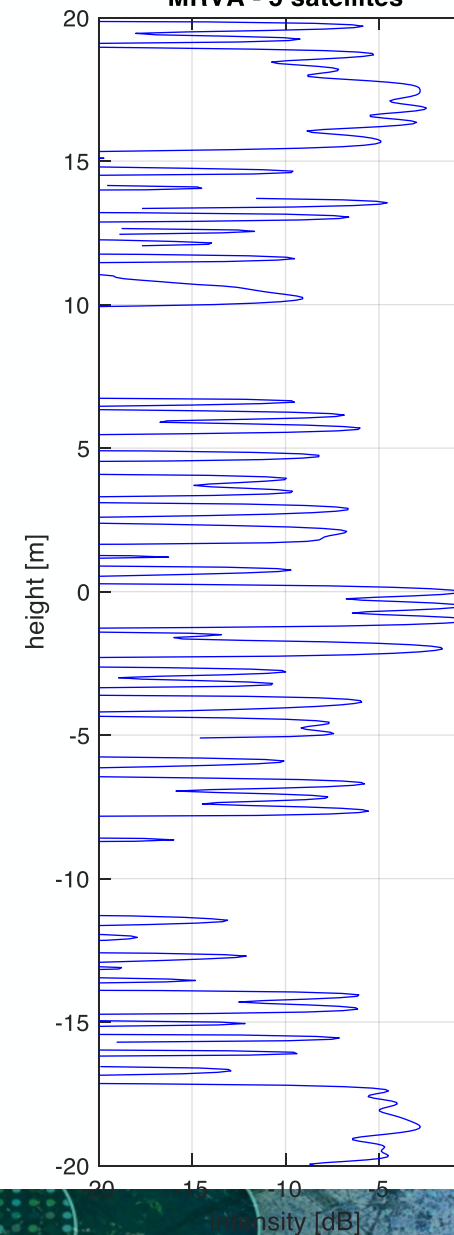
MRWI - 5 satellites



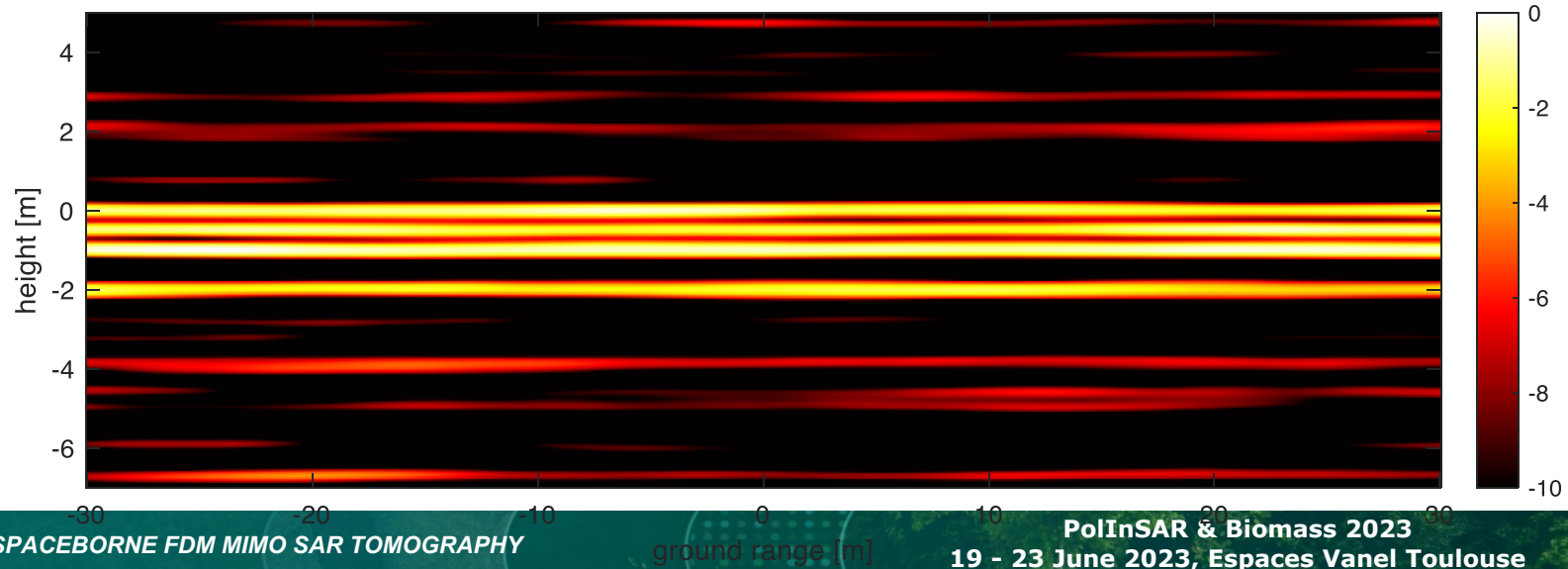
MRVA - 5 satellites



MRVA - 5 satellites



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The work in this paper provides a theoretical ground to the possibility of building a high-resolution spaceborne tomographic SAR system while using a reasonably low number of satellites.

- Distributed MIMO technology is a strict requirement
- FDM is a viable alternative to complex hardware solutions involving onboard beamforming and multiplication of channels on individual satellites

Formation deployment was discussed by extending the concept of MRVA to the case of distributed scatterers, resulting in a new criterion referred to as MRWI

- Numerical simulations support the possibility to deploy a formation of 4 or 5 satellites to provide the equivalent of 16 and 25 monostatic orbits.
- Such figures are comparable to the best airborne and ground-based systems available as of today, and indicate the concrete possibility to image the vertical structure of natural targets from space at fine resolution.

A **roadmap** for future development of this concept should address technological issues, orbital control, and the development of methods specific to FDM MIMO to recover clock synchronization and fine positioning