

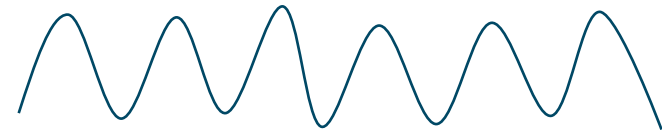
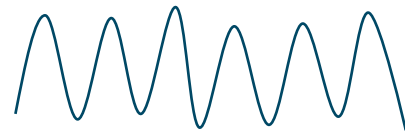
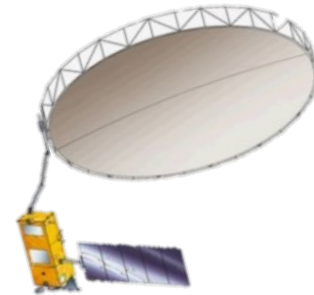
# MULTIFREQUENCY COHERENCE TOMOGRAPHY IN FORESTS

Roman Guliaev, Matteo Pardini, Konstantinos Papathanassiou  
German Aerospace Center (DLR)



# Multifrequency data

- Different sensors
- Frequencies
- Acquisition time



Time



# AFRISAR 2016 and GABONX 2023



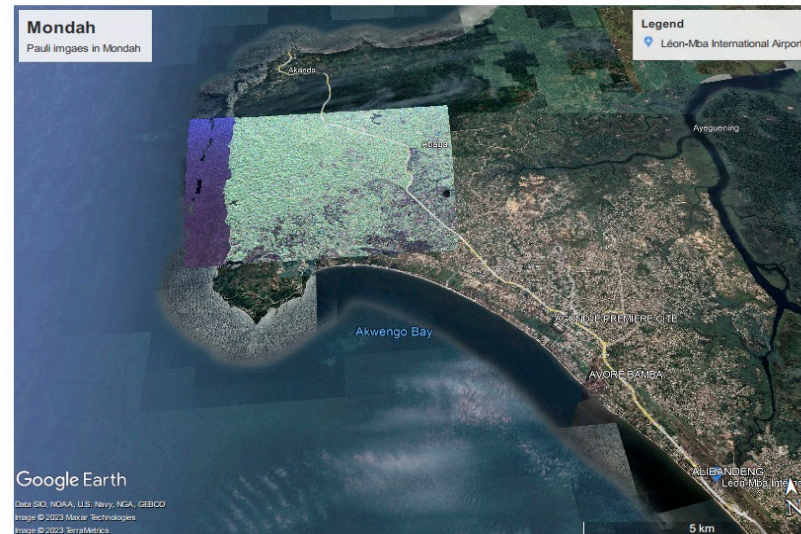
Image credit: Ralf Horn



Do 228, FSAR

# AFRISAR 2016 and GABONX 2023

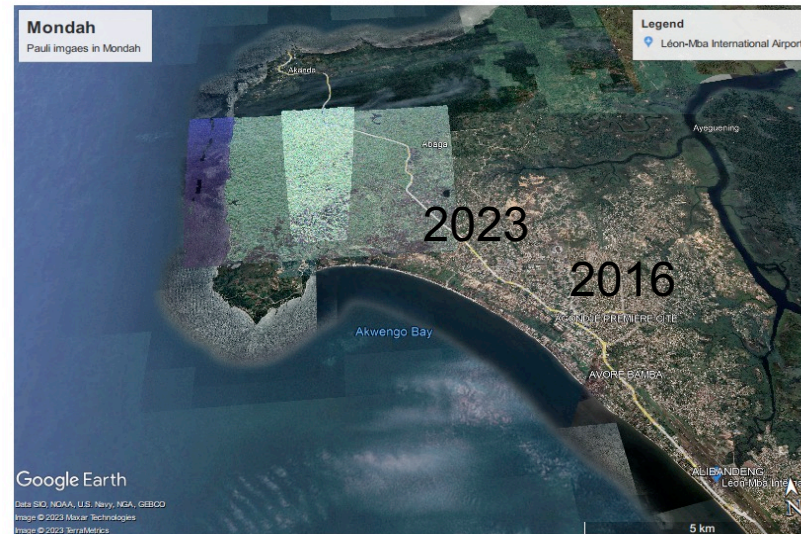
- L-, P- bands
- 12 tomographic acquisitions



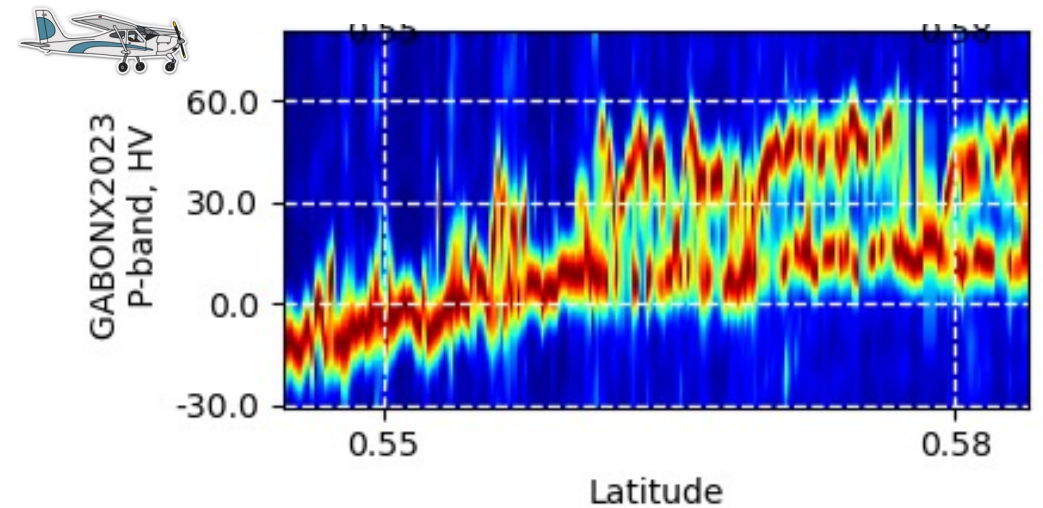
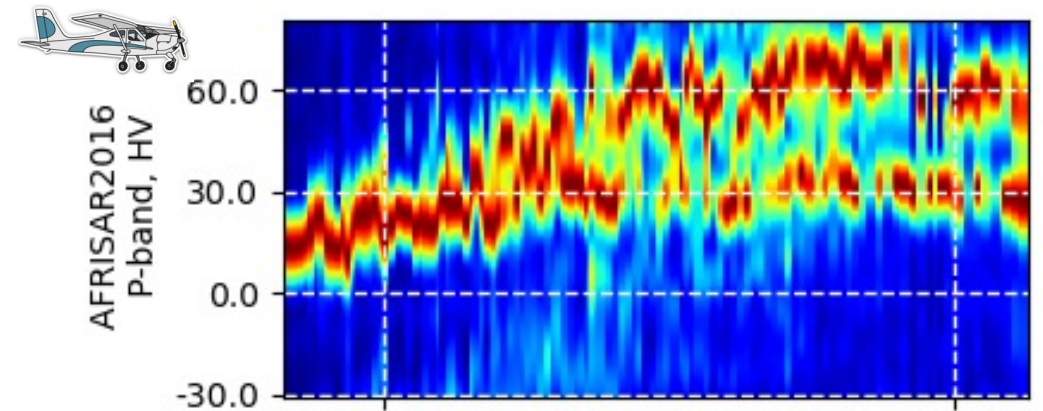
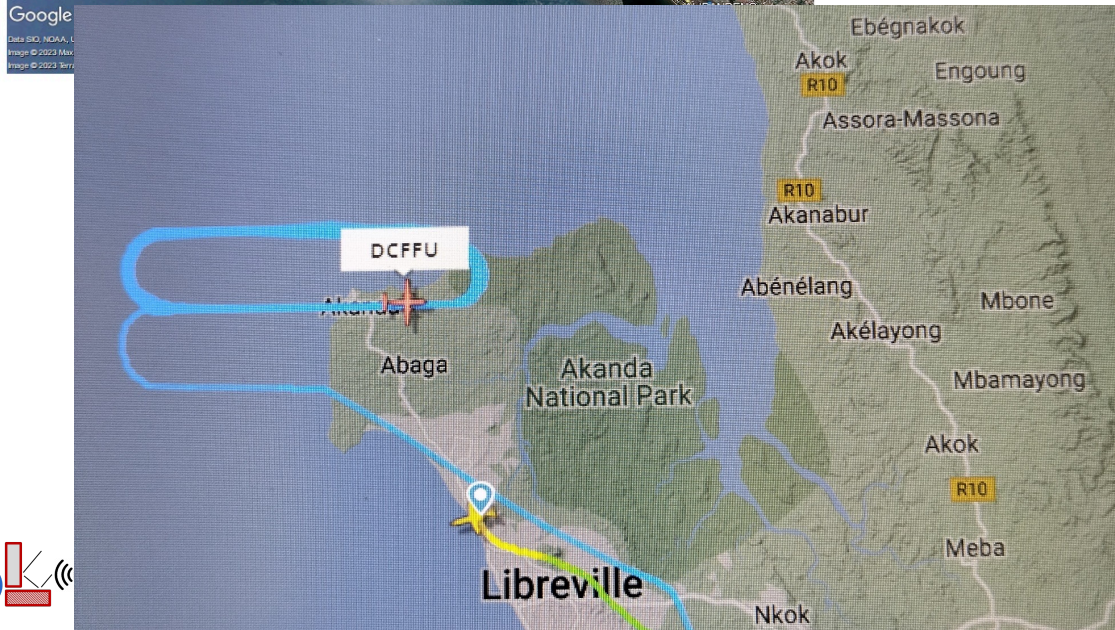
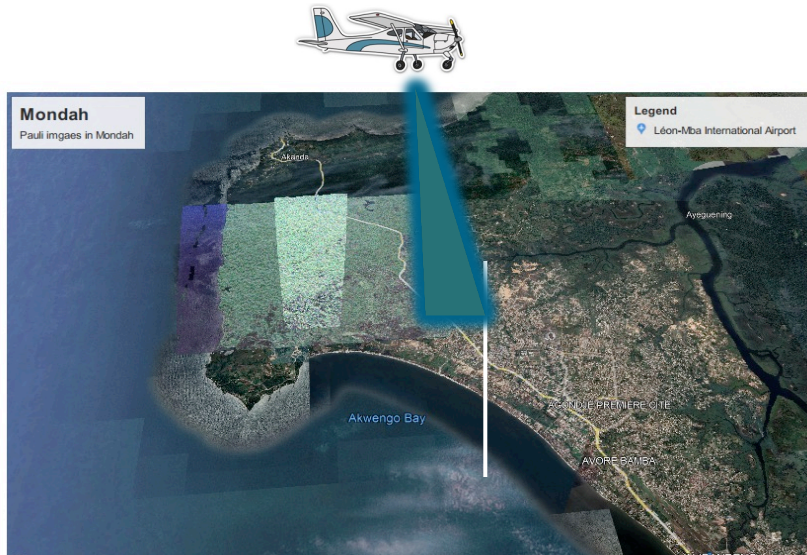


# AFRISAR 2016 and GABONX 2023

- L-, P- bands
- 12 tomographic acquisitions



# Beamforming profiles P-band, AFRISAR 2016 and GABONX2023

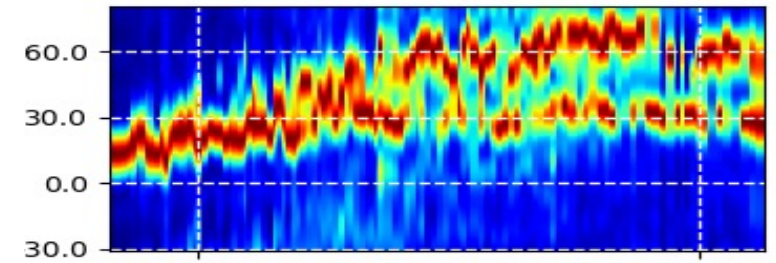




# Detecting Change between 2016 and 2023



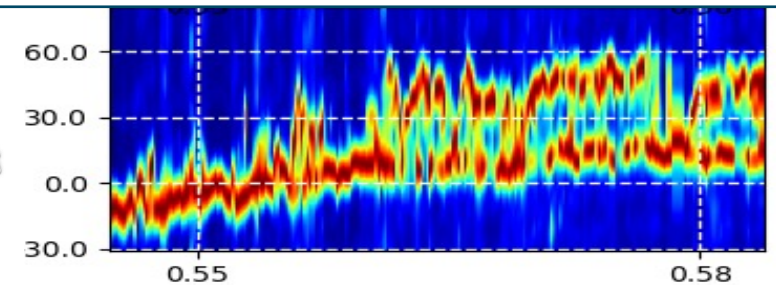
2016



- Need info between 2016/2023
- TanDEM-X filling
- But no tomographic acquisitions
- Using Coherency Tomography



2023



# Coherence Tomography

$$\gamma_v = e^{i\phi_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz}$$

$$f(z) \approx f_0(z) + A_1 f_1(z) + A_2 f_2(z)$$

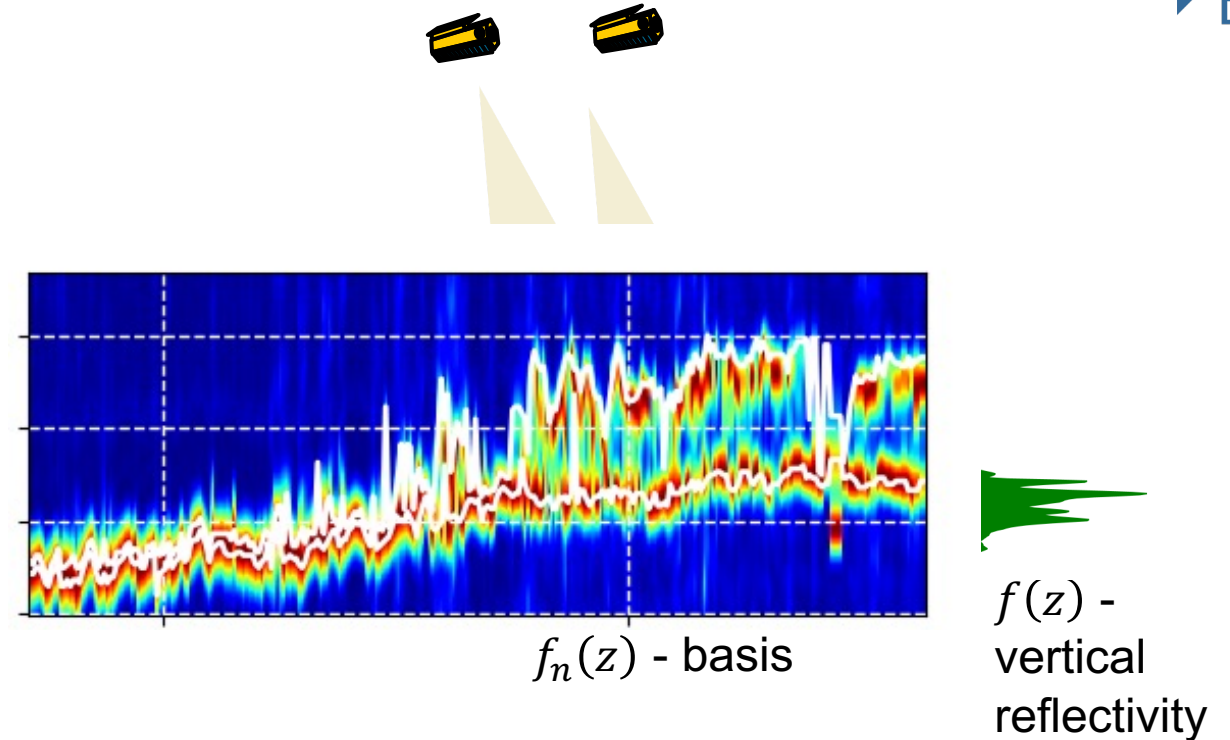
$$\gamma_v = e^{i\phi_0} \frac{\int_0^{h_v} [f_0(z) + A_1 f_1(z) + A_2 f_2(z)] e^{ik_z z} dz}{\int_0^{h_v} [f_0(z) + A_1 f_1(z) + A_2 f_2(z)] dz}$$

2 linear equations => find  $A_1$  and  $A_2$

$$A_n = \int_0^{h_v} f_n(z) f(z) dz$$

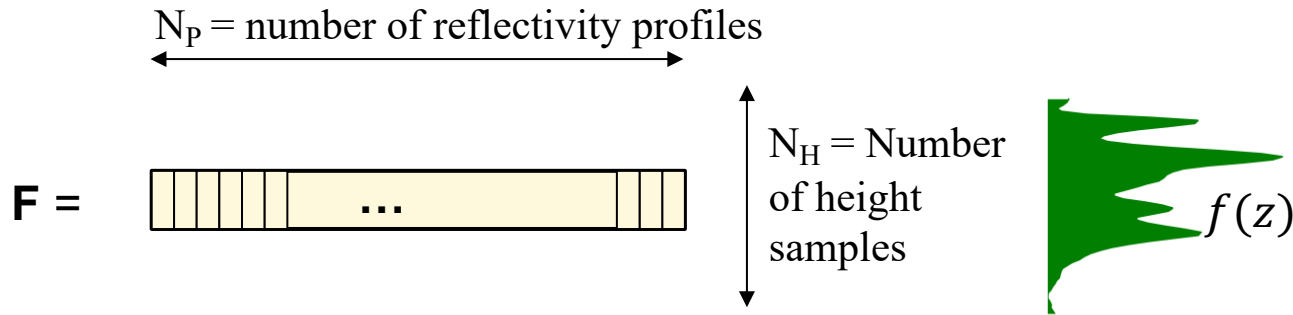
$\phi_0$  and  $h_v$  can be derived from P band tomography

S. Cloude „Polarization Coherence Tomography“ , 2006





# Deriving the basis for Coherence Tomography



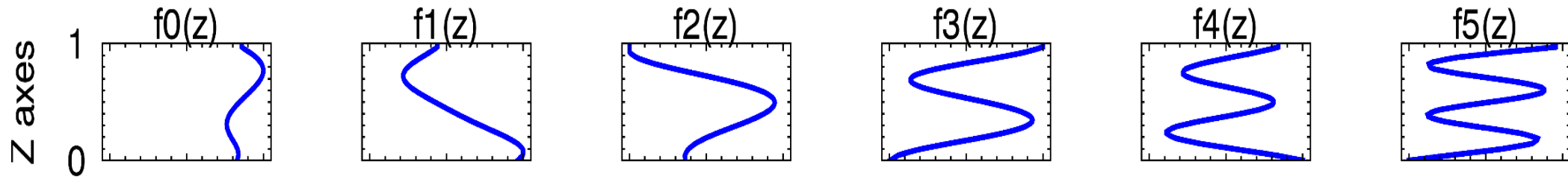
Profile covariance matrix :

$$\mathbf{C} = \mathbf{F} \mathbf{F}^T = \begin{bmatrix} \dots & c_{ij} & \dots \end{bmatrix}$$

$N_H$

$$c_{ij} = \frac{1}{N_p} \sum_{s=1}^{N_p} F_{is} F_{js}$$

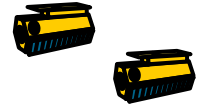
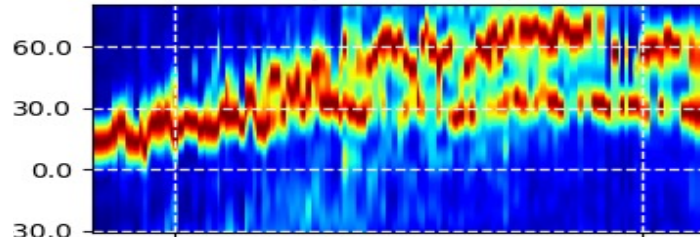
## P band basis



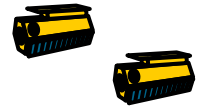
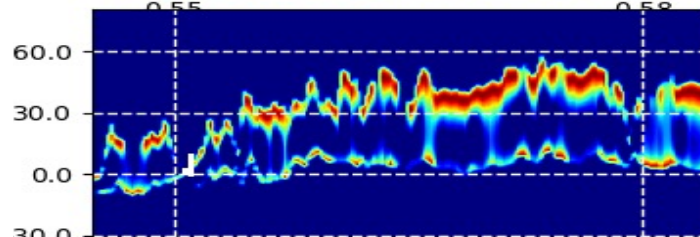
# Coherence Tomography, P band



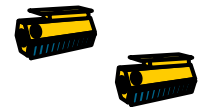
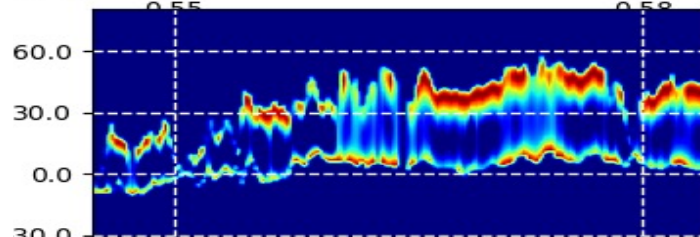
2016



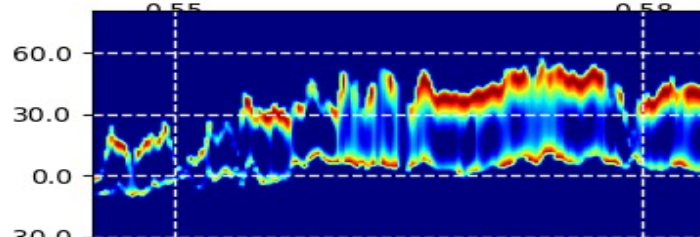
2015



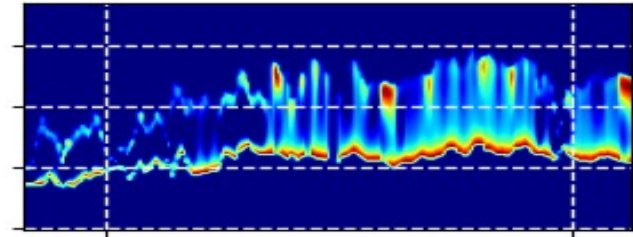
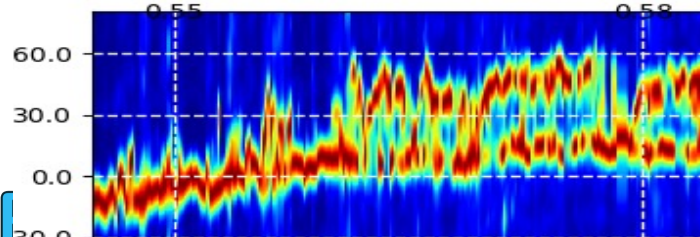
2017



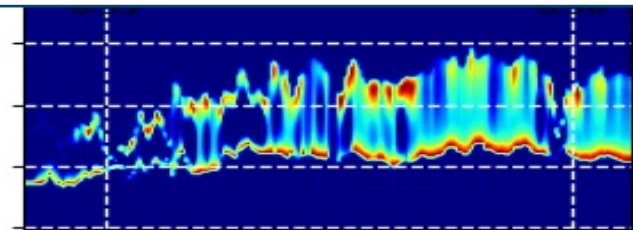
2019



2023

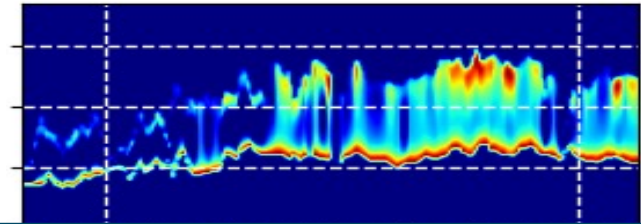
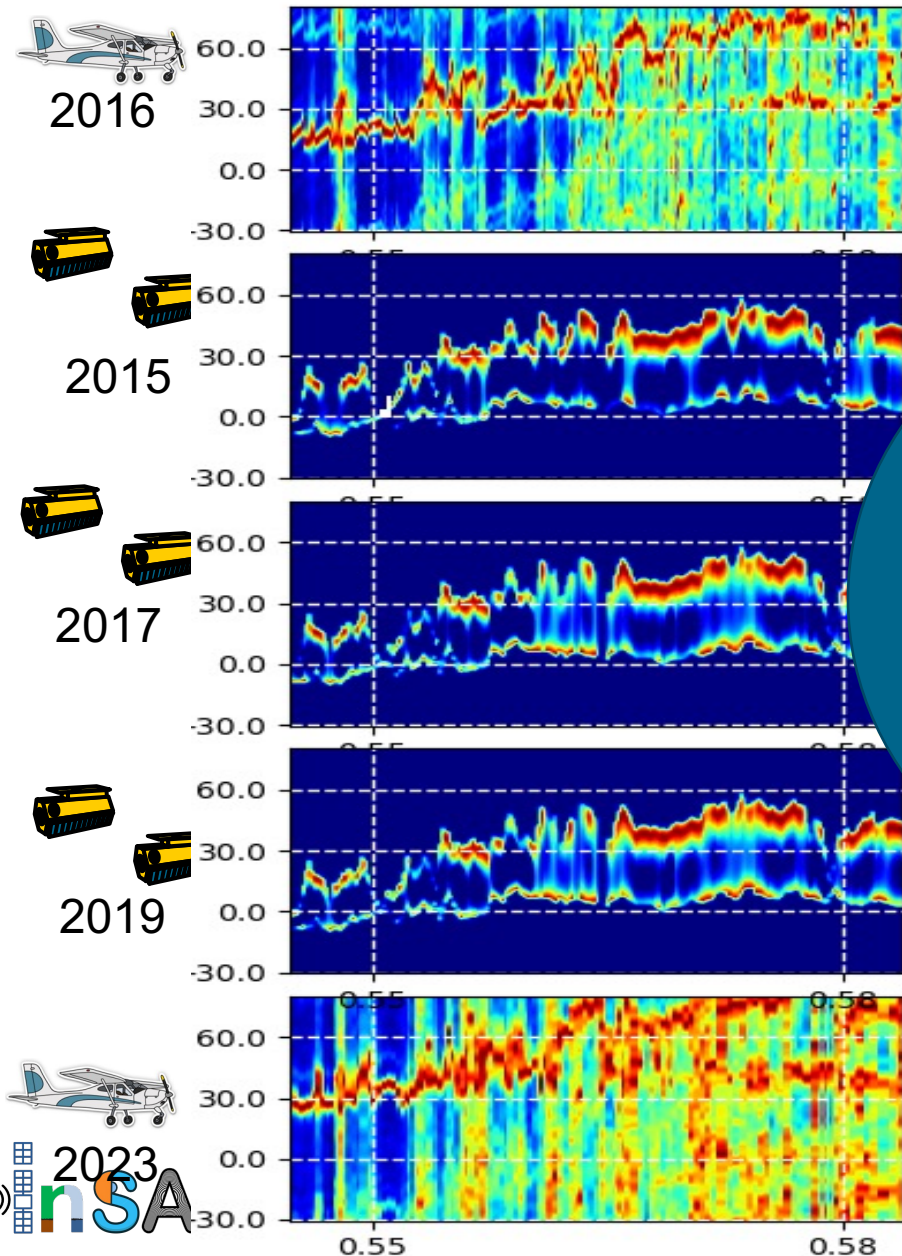


Both P- and X-band  
Coherence Tomography  
profiles



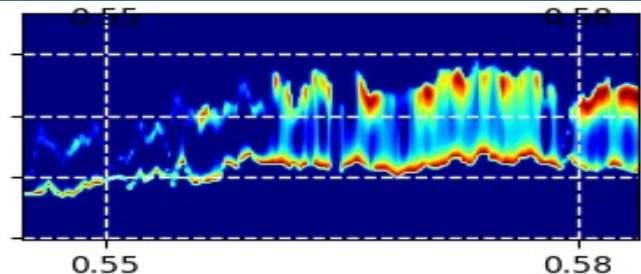


# L band



Same P band basis across frequencies!

L- and X-band  
interference tomography  
results



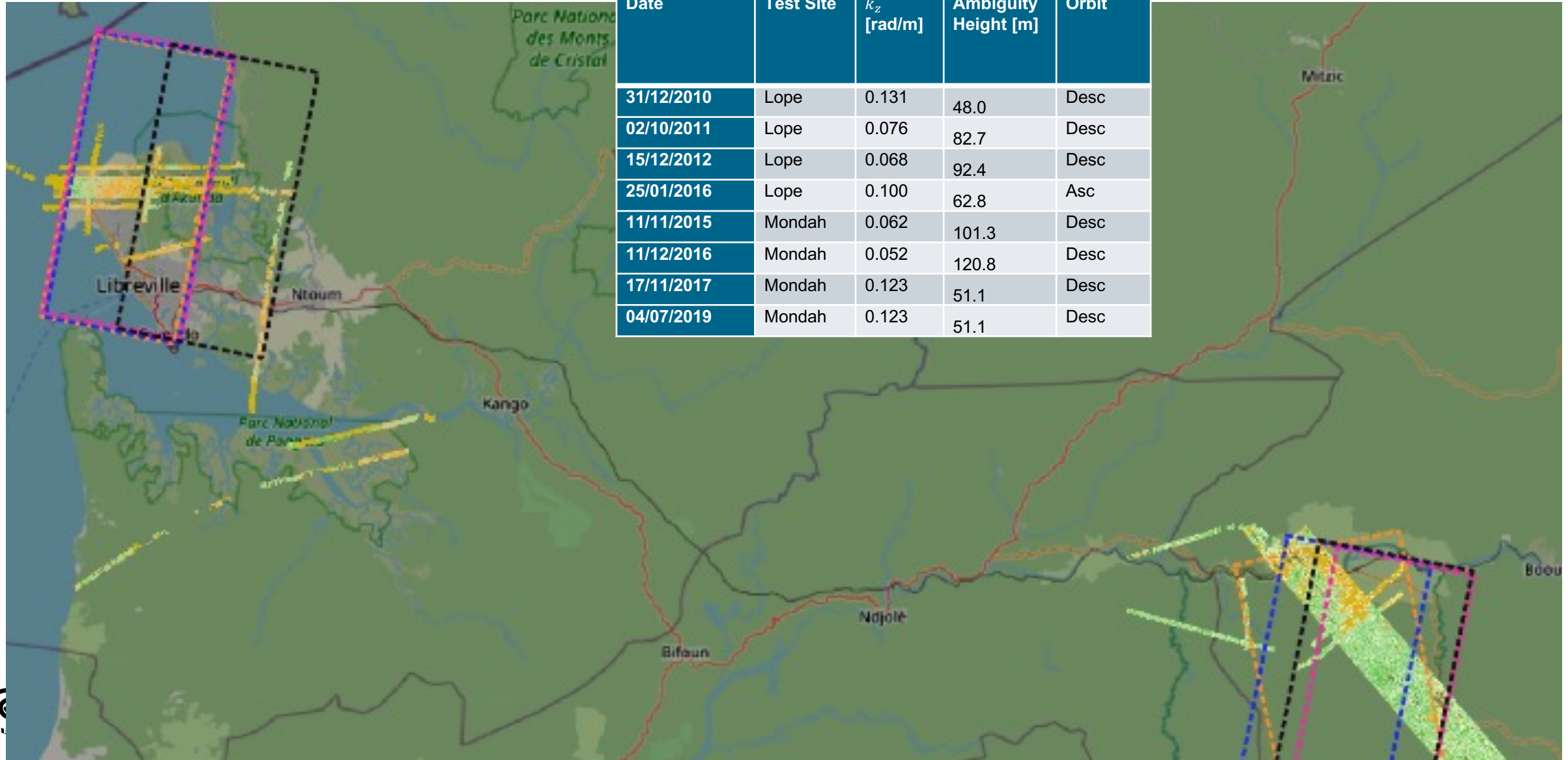
# Caveats

- Method suffers from temporal decorrelations
- Ground phase errors
- Depends on the used basis
- Can produce negative profiles





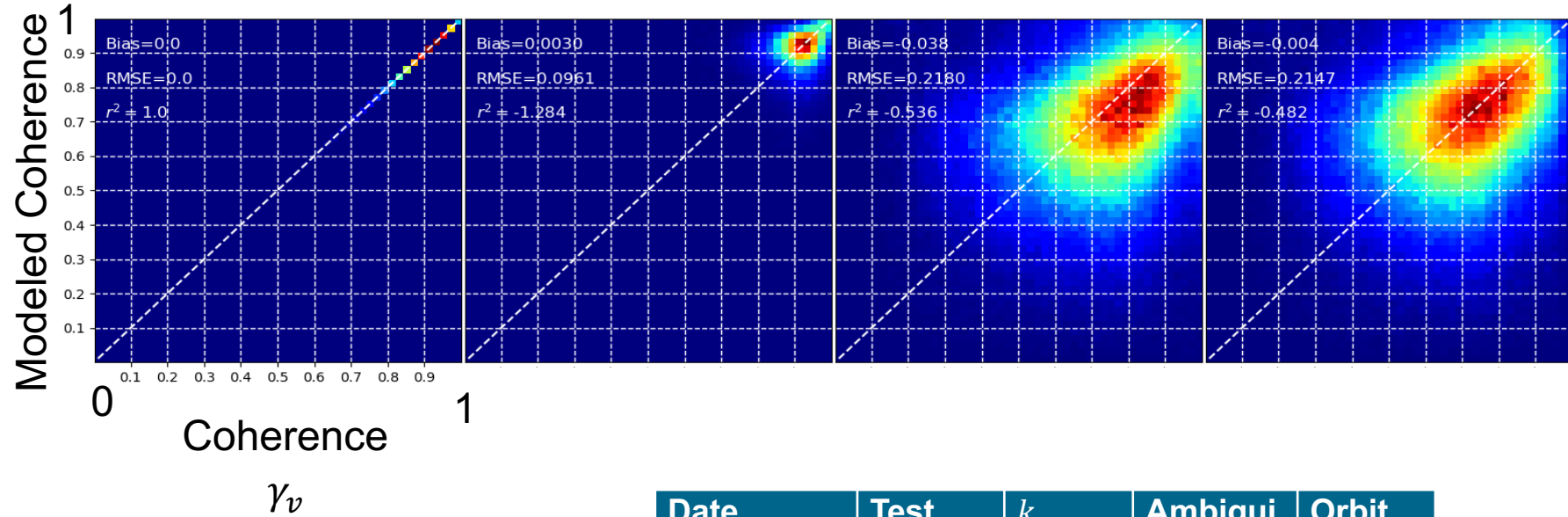
# TanDEM-X acquisitions



Date	Test Site	$k_z$ [rad/m]	Ambiguity Height [m]	Orbit
31/12/2010	Lope	0.131	48.0	Desc
02/10/2011	Lope	0.076	82.7	Desc
15/12/2012	Lope	0.068	92.4	Desc
25/01/2016	Lope	0.100	62.8	Asc
11/11/2015	Mondah	0.062	101.3	Desc
11/12/2016	Mondah	0.052	120.8	Desc
17/11/2017	Mondah	0.123	51.1	Desc
04/07/2019	Mondah	0.123	51.1	Desc

# Quality assessment

$$\gamma_v^{mod}(\kappa_z) = \frac{\int_0^1 f_{PCT}(z) \exp(i\kappa_z h_v^{ref} z) dz}{\int_0^1 f_{PCT}(z) dz}$$

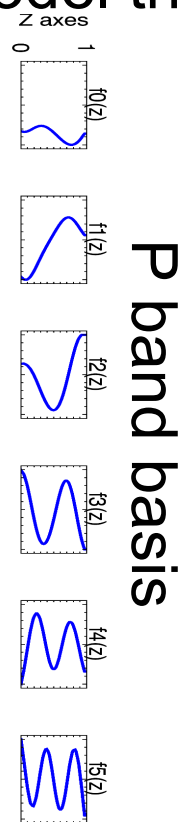
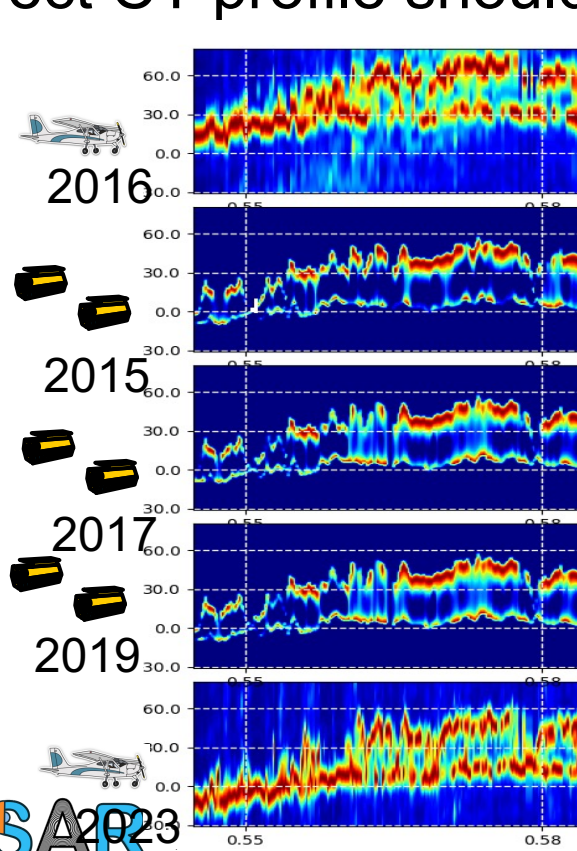


Date	Test Site	$k_z$ [rad/m]	Ambiguity Height [m]	Orbit
11/11/2015	Mondah	0.062	101.3	Desc
11/12/2016	Mondah	0.052	120.8	Desc
17/11/2017	Mondah	0.123	51.1	Desc
04/07/2019	Mondah	0.123	51.1	Desc

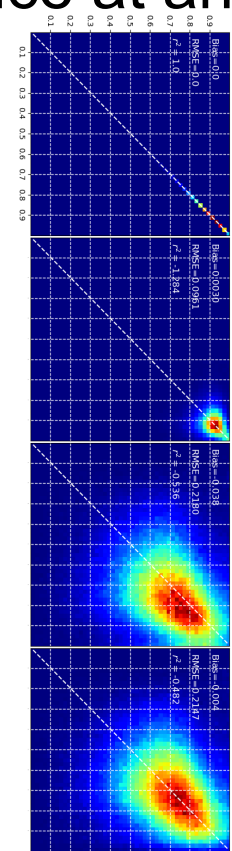


# Summary

- The InSAR profiles (Coherence Tomography) can facilitate the interpretation of changes between TomoSAR profiles separated in time
- Same basis for different frequencies can be applied
- Correct CT profile should correctly model the coherence at any baseline

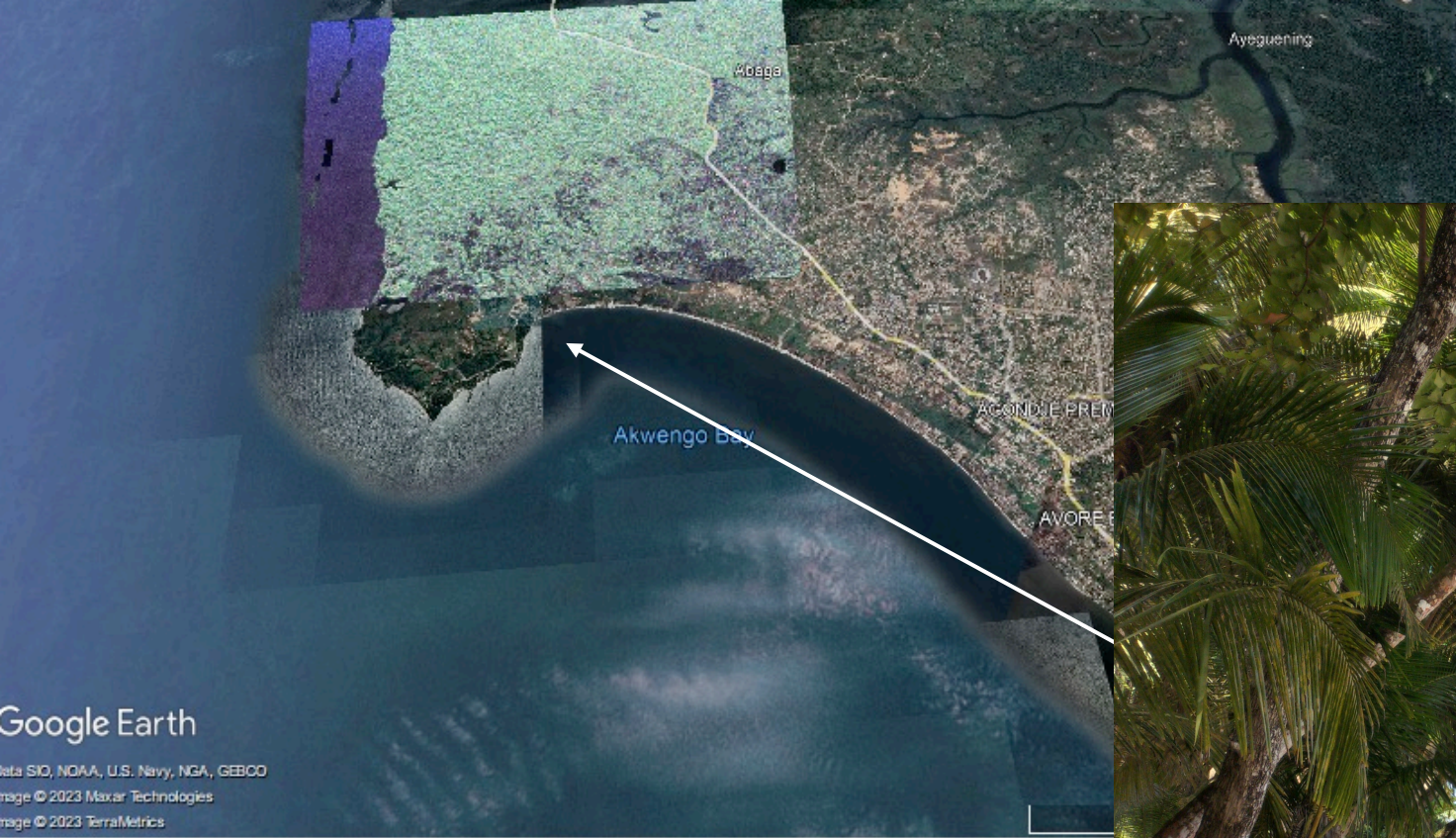


P band basis



Validation





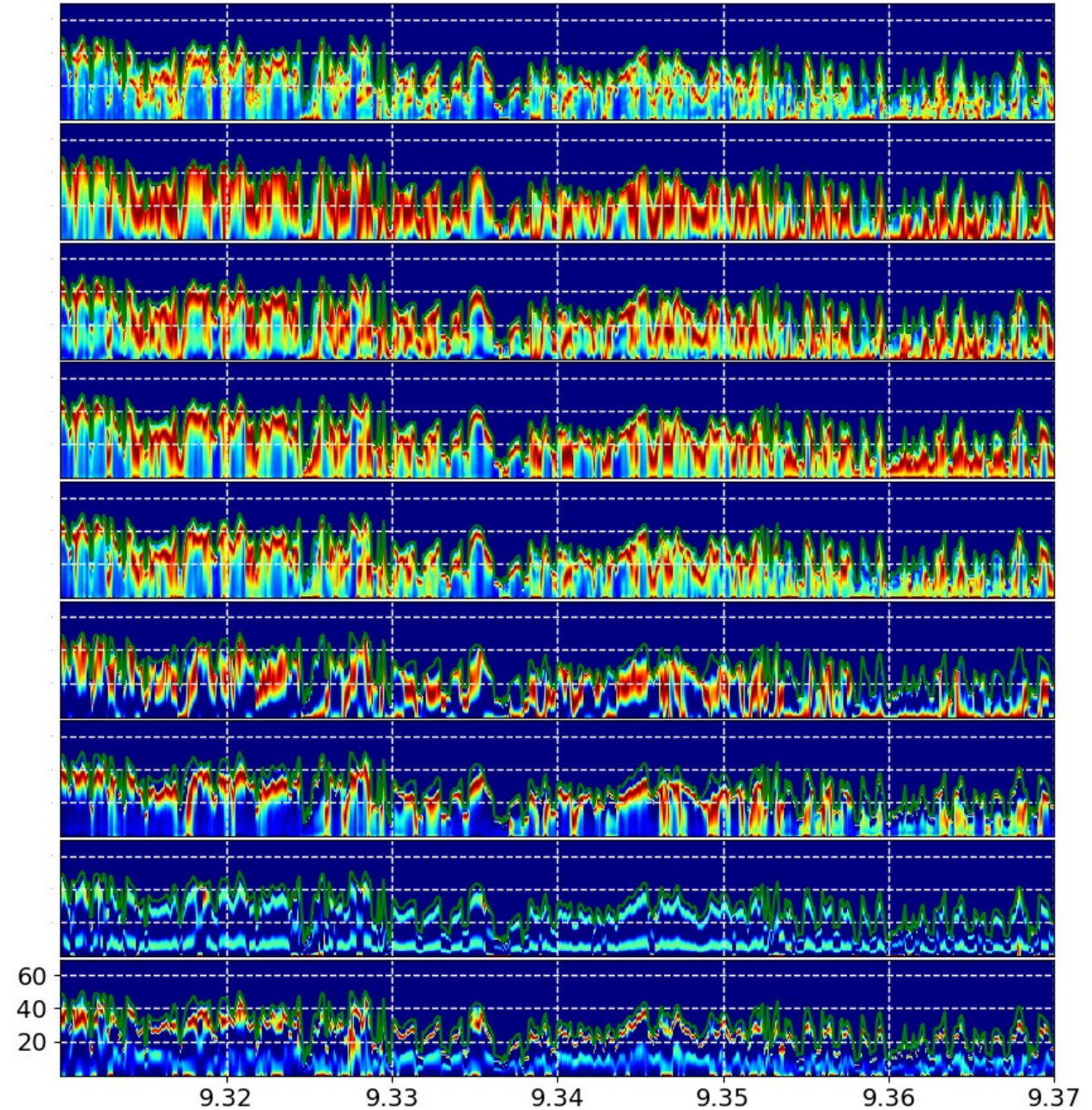
Google Earth

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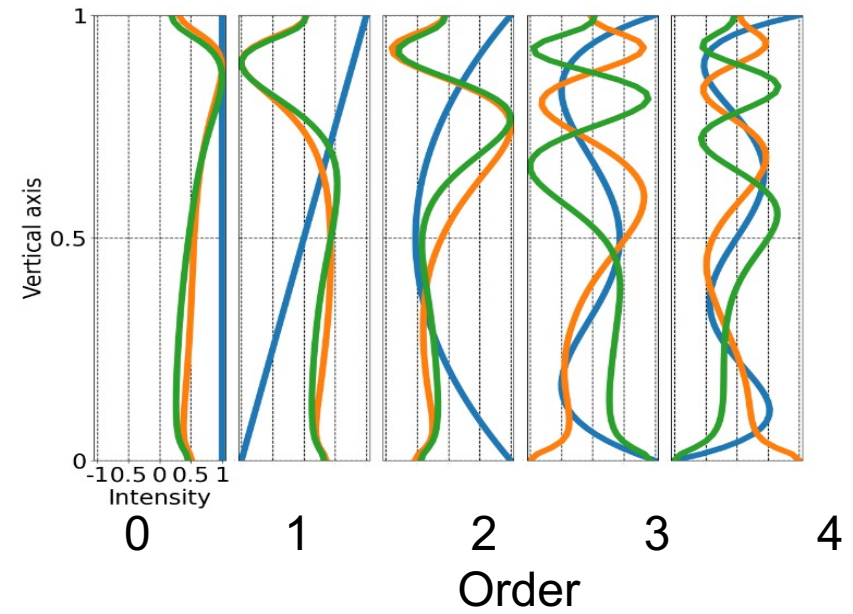
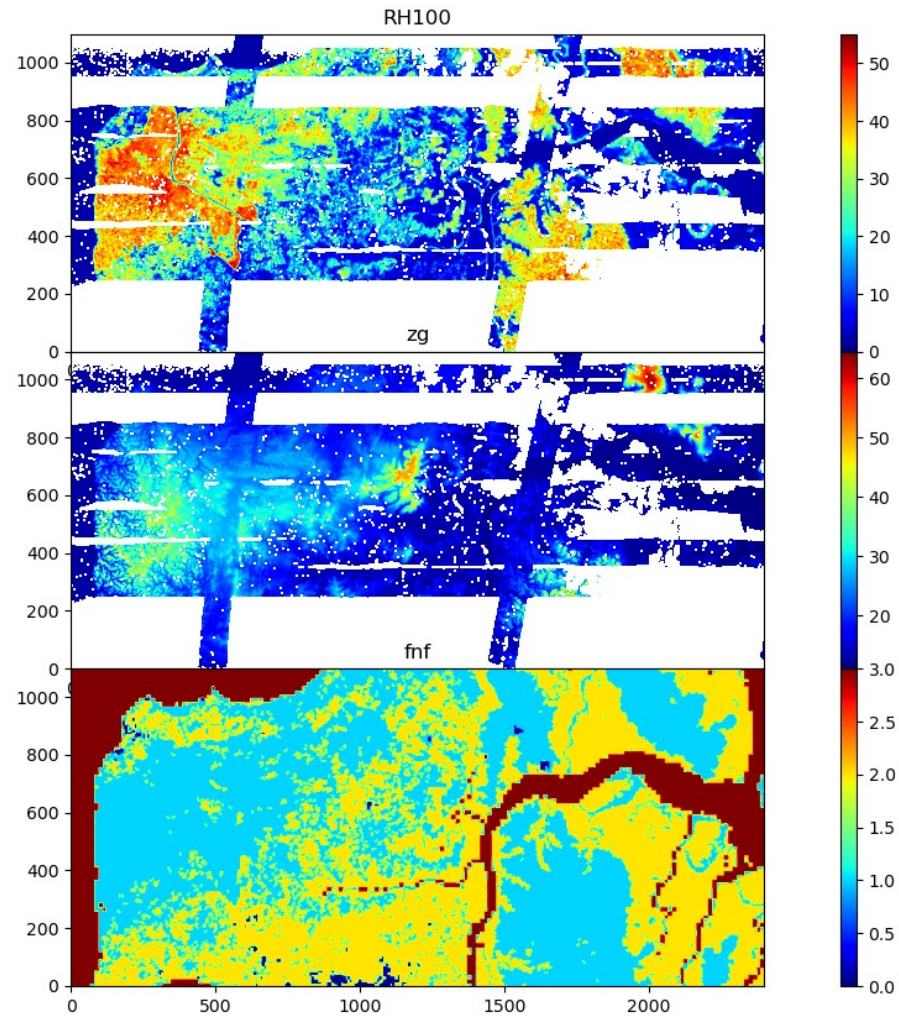




- Lidar profile
- Lidar profile, decomposed on 3 coef. of Legendre
- Lidar profile, decomposed on 3 coef. of Eigenbasis
- Lidar profile, decomposed on 5 coef. of Legendre
- Lidar profile, decomposed on 5 coef. of Eigenbasis
- Single baseline, X-band on Legendre
- Single baseline, X-band on Eigenbasis
- Dual baseline, X-band on Legendre
- Dual baseline, X-band on Eigenbasis



# Lidar data

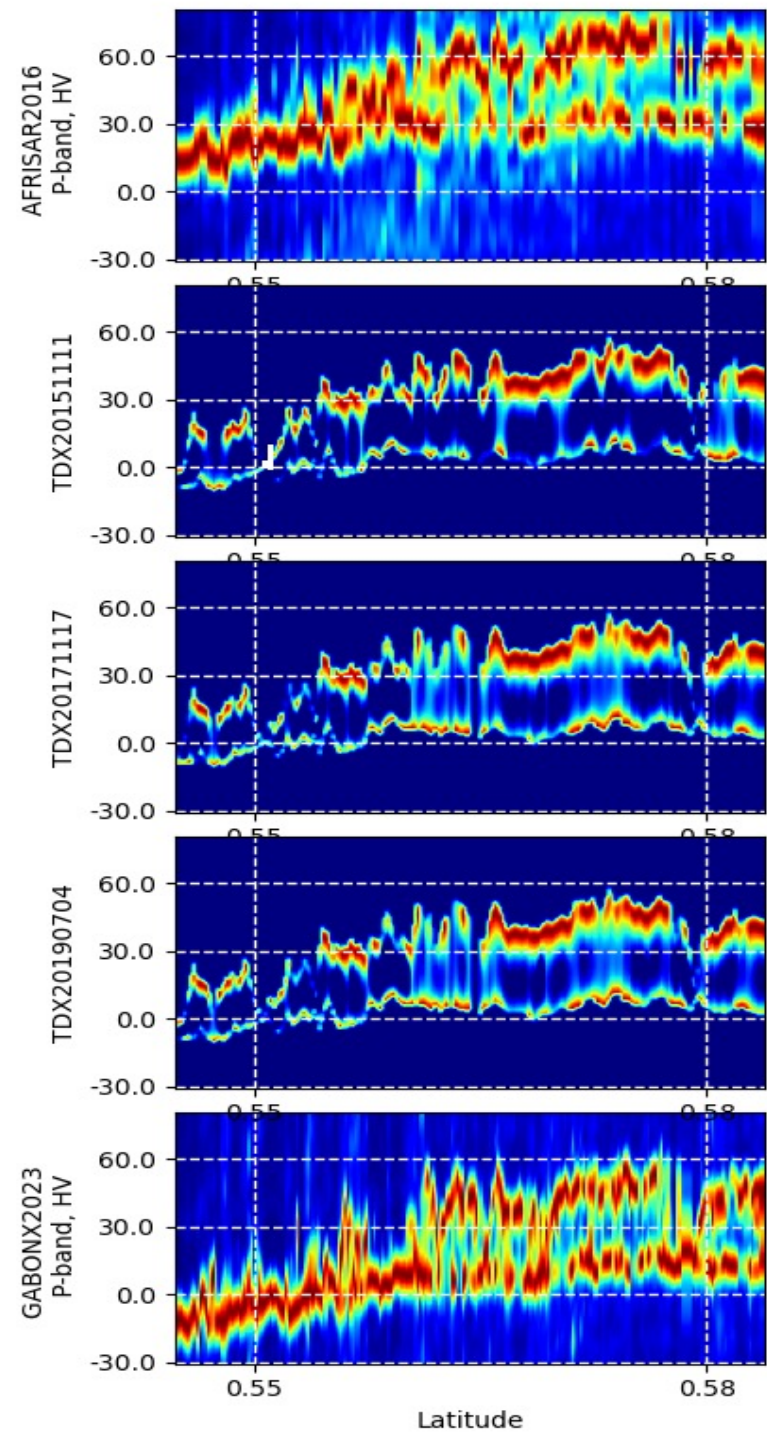


$f_n(z)$  - basis

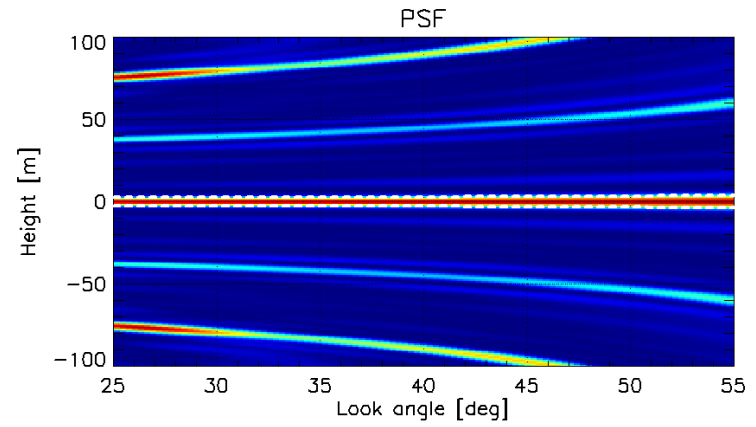
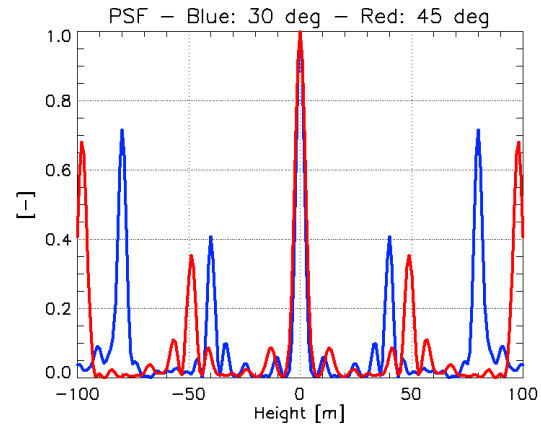
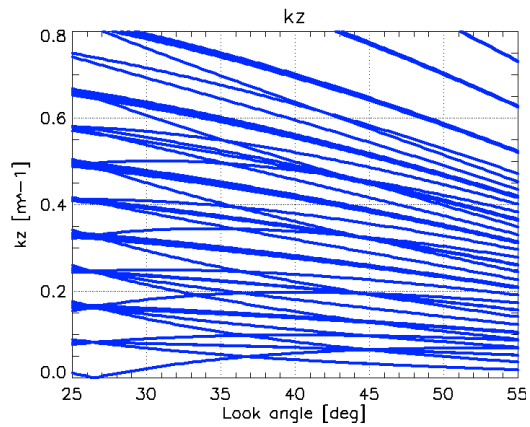
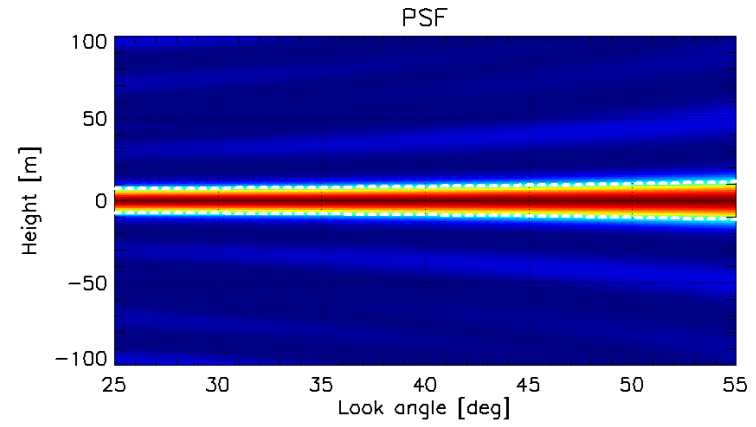
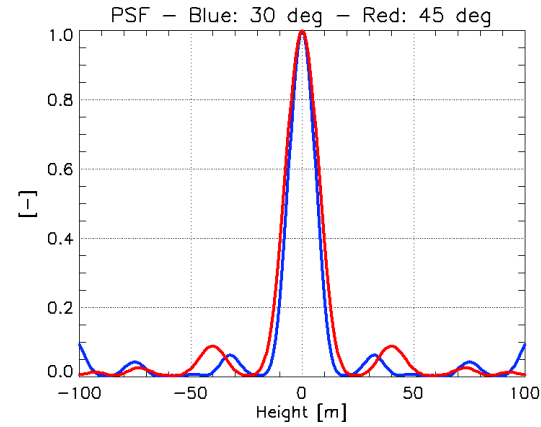
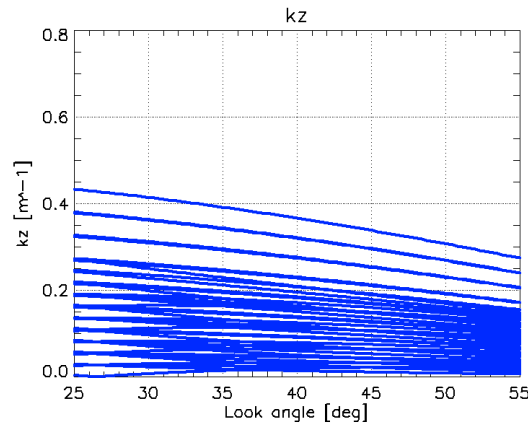




# Profile propagation across frequencies and time



# Tomographic baseline distribution



# General form of linear system

$$FA = B$$

$$F = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \text{Im} \{F_n (k_z^k h_v) - \gamma_v^k F_n'\} & \dots \\ \dots & \text{Re} \{F_n (k_z^k h_v) - \gamma_v^k F_n'\} & \dots \\ \dots & \dots & \dots \end{bmatrix}$$

$$B = \begin{bmatrix} \dots & \dots \\ \text{Im} \{ \gamma_v^k F_0' - F_0 (k_z^k h_v) \} & \dots \\ \text{Re} \{ \gamma_v^k F_0' - F_0 (k_z^k h_v) \} & \dots \\ \dots & \dots \end{bmatrix}$$

$$A = \begin{bmatrix} \dots \\ A_n \\ \dots \end{bmatrix}$$

$$F \in \mathbb{R}(2K \times N)$$

$$B \in \mathbb{R}(2K)$$

$$A \in \mathbb{R}(N)$$

$$\gamma_v^k = \frac{\int_0^1 [f_0(z) + \dots A_n f_n(z) + \dots] e^{ik_z^k h_v z} dz}{\int_0^1 [f_0(z) + \dots A_n f_n(z) + \dots] dz}$$

K – number of baselines  $k_z^k$  with  $\gamma_v^k$

N - number of basis coefficients

$$F_n(k_z^k h_v) = \int_0^1 e^{ik_z^k h_v z} f_n(z) dz$$

$$A = F^{-1}B$$

$$F_n' = \int_0^1 f_n(z) dz$$

SVD solution:

$$F = U\Sigma V^T$$

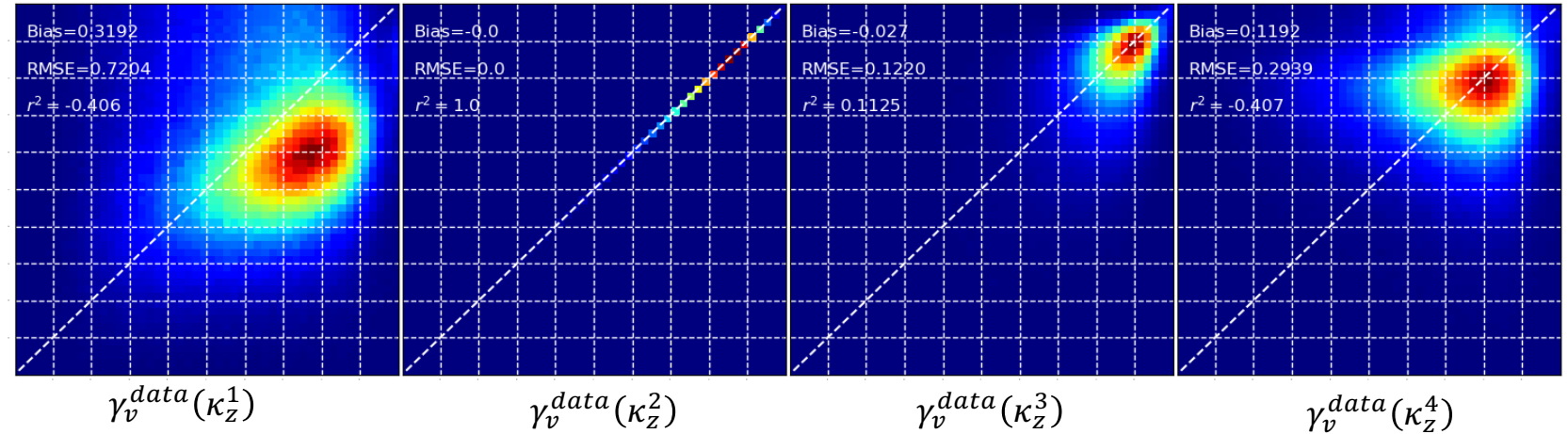
$$A = V^T \Sigma U \quad B$$



# Single baseline, Lope

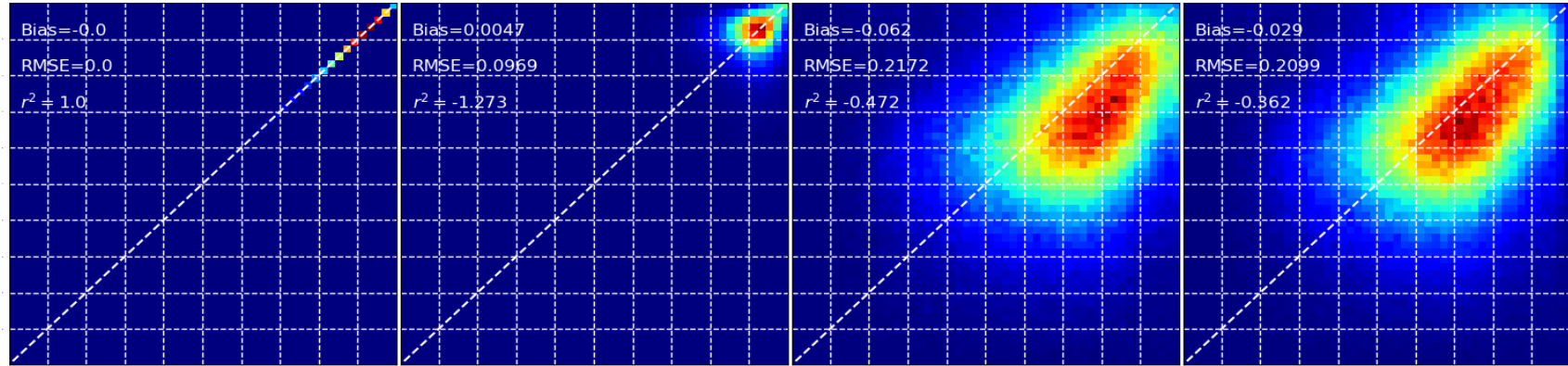
- Legendre

$$\gamma_v^{mod}(\kappa_z) = \frac{\int_0^1 f_{PCT}(z) \exp(ik_z h_v^{ref} z) dz}{\int_0^1 f_{PCT}(z) dz}$$

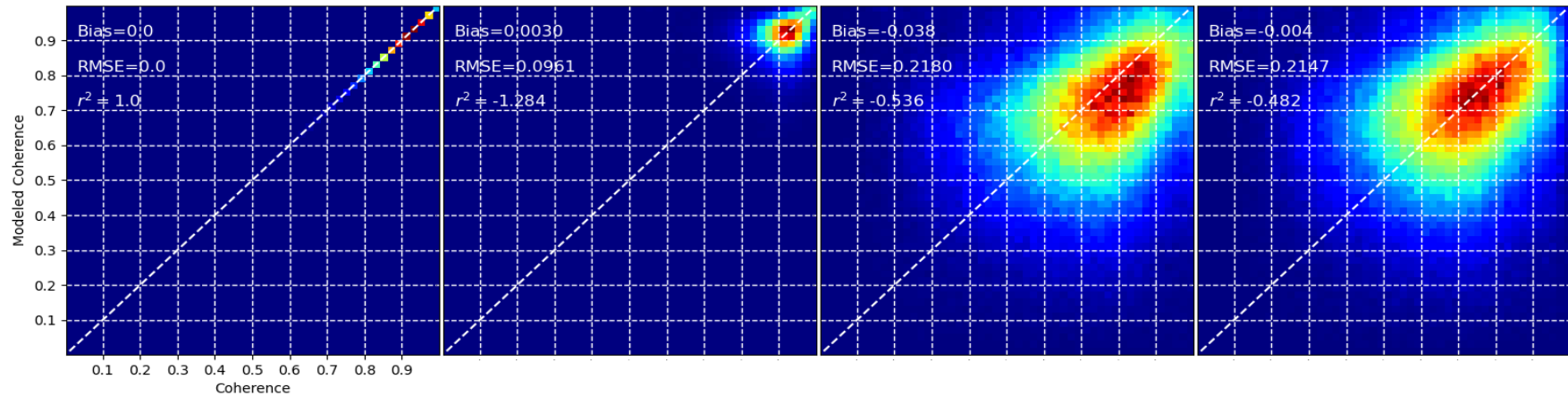


# Single baseline Model

- Legendre

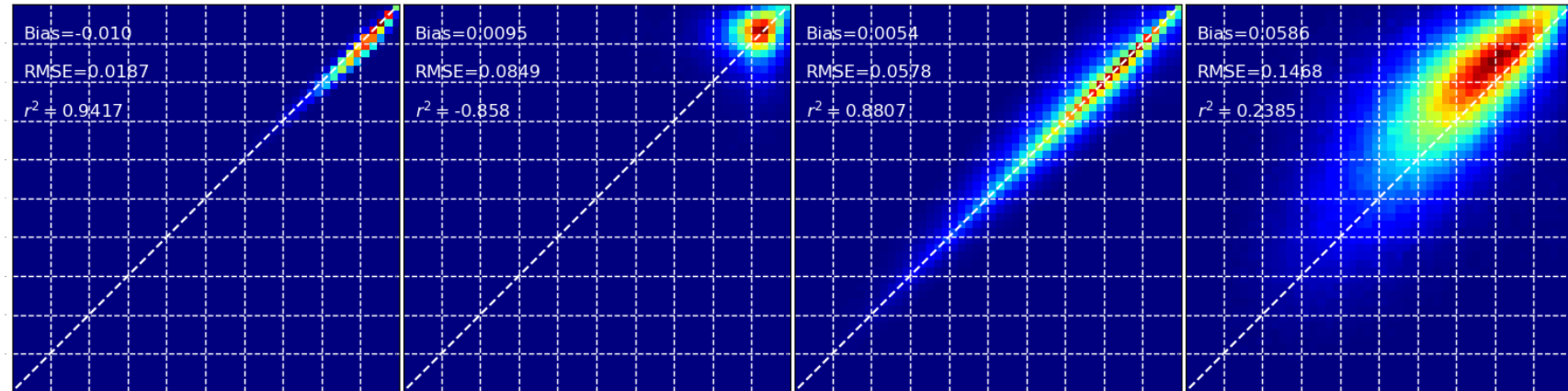


- Eigenbasis



# Dual baseli

- Legendre



- Eigenbasis

